SAUDI STANDARD
NO. ١٨٦٦١

POWER TRANSFORMERS – APPLICATION GUIDE

SAUDI ARABIAN STANDARDS ORGANIZATION

THIS DOCUMENT IS A DRAFT SAUDI STANDARD CIRCULATED FOR COMMENTS. IT IS, THEREFORE, SUBJECT TO CHANGE AND MAY NOT BE REFERRED TO AS A SAUDI STANDARD UNTIL APPROVED BY THE BOARD OF DIRECTORS.
CONTENTS

FOREWORD .......................................................................................................................... 2

Clause

1 General ................................................................................................................................ 3

2 Characteristic properties of different three-phase winding combinations and magnetic circuit designs ........................................................................................................... 4

3 Characteristic properties and application of auto-connected transformers ........................................... 9

4 Zero-sequence properties – neutral load current and earth fault conditions, magnetic saturation and inrush current ........................................................................................ 13

5 Calculation of short-circuit currents in three-winding, three-phase transformers (separate winding transformers and auto-connected transformers) with earthed neutrals ................................................................. 26

6 Parallel operation of transformers in three-phase systems ......................................................... 81

7 Calculation of voltage drop for a specified load, three-winding transformer load loss ......................... 93

8 Specification of rated quantities and tapping quantities ..................................................................... 120

9 Convertor applications with standard transformers ......................................................................... 147

10 Guide to the measurement of losses in power transformers .......................................................... 161

Annex A – Basic relations for single-phase and two-phase earth faults ........................................... 160
FOREWORD

The Saudi Standards and Quality Organization (SASO) has adopted the International Standard IEC 6700-8 “Power Transformers – Application Guide” issued by the International Electrotechnical Commission (IEC). It has been adopted without any technical modifications with a view to its approval as a Saudi standard.
POWER TRANSFORMERS – APPLICATION GUIDE

1 General

1.1 Scope and object

This Standard applies to power transformers complying with the series of publications IEC 67006.

It is intended to provide information to users about:

- certain fundamental service characteristics of different transformer connections and magnetic circuit designs, with particular reference to zero-sequence phenomena;
- system fault currents in transformers with YNynd and similar connections;
- parallel operation of transformers, calculation of voltage drop or rise under load, and calculation of load loss for three-winding load combinations;
- selection of rated quantities and tapping quantities at the time of purchase, based on prospective loading cases;
- application of transformers of conventional design to convertor loading;
- measuring technique and accuracy in loss measurement.

Part of the information is of a general nature and applicable to all sizes of power transformers. Several chapters, however, deal with aspects and problems which are of the interest only for the specification and utilization of large high-voltage units.

The recommendations are not mandatory and do not in themselves constitute specification requirements.

Information concerning loadability of power transformers is given in IEC 6304, for oil-immersed transformers, and IEC 6405, for dry-type transformers.

Guidance for impulse testing of power transformers is given in IEC 6732.

1.2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the editions indicated were valid. All normative documents are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent edition of the normative documents indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.


IEC 67006: Power transformers

Characteristic properties of different three-phase winding combinations and magnetic circuit designs

This chapter is an overview of the subject. Additional information is given in clause 4 on zero-sequence properties.

2.1 Y, D, and Z-connected windings

There are two principal three-phase connections of transformer windings: star (Y-connection) and delta (D-connection). For special purposes, particularly in small power transformers, another connection named zigzag or Z is also used. Historically, several other schemes have been in use (such as "truncated delta", "extended delta", "T-connection", "V-connection", etc.). While such connections are used in transformers for special applications, they no longer appear in common power transmission systems.

2.1.1 Advantages of a Y-connected winding

This type of winding:
- is more economical for a high-voltage winding;
- has a neutral point available;
- permits direct earthing or earthing through an impedance;
- permits reduced insulation level of the neutral (graded insulation);
- permits the winding taps and tapchanger to be located at the neutral end of each phase;
- permits single-phase loading with neutral current (see 2.4 and 4.8).
\section{Advantages of a D-connected winding}

This type of winding:

- is more economical for a high-current, low-voltage winding;
- in combination with a star-connected winding, reduces the zero-sequence impedance in that winding.

\section{Advantages of a Z-connected winding}

This type of winding:

- permits neutral current loading with inherently low zero-sequence impedance. (It is used for earthing transformers to create an artificial neutral terminal of a system);
- reduces voltage unbalance in systems where the load is not equally distributed between the phases.

\section{Characteristic properties of combinations of winding connections}

The notation of winding connections for the whole transformer follows the conventions in IEC 67006-1, clause 6.

This subclause is a summary of the neutral current behaviour in different winding combinations. Such conditions are referred to as having "zero-sequence components" of current and voltage. This concept is dealt with further in clauses 4 and 5.

The statements are also valid for three-phase banks of single-phase transformers connected together externally.

\subsection{YNyn and YNauto}

Zero-sequence current may be transformed between the windings under ampere-turn balance, meeting low short-circuit impedance in the transformer. System transformers with such connections may in addition be provided with delta equalizer winding (see 2.7.4 and 8.4).

\subsection{YNy and Yyn}

Zero-sequence current in the winding with earthed neutral does not have balancing ampere-turns in the opposite winding, where the neutral is not connected to earth. It therefore constitutes a magnetizing current for the iron core and is controlled by a zero-sequence magnetizing impedance. This impedance is high or very high, depending on the design of the magnetic circuit (see 3.2). The symmetry of the phase-to-neutral voltages will be affected and there may be limitations for the allowable zero-sequence current caused by stray-flux heating (see 1.8).

\subsection{YNd, Dyn, YNyd (loadable tertiary) or YNy + d (non-loadable delta equalizer winding)}

Zero-sequence current in the star winding with earthed neutral causes compensating circulating current to flow in the delta winding. The impedance is low, approximately equal to the positive-sequence short-circuit impedance between the windings.
If there are two star windings with earthed neutrals (including the case of auto-connection with common neutral), there is a three-winding loading case for zero-sequence current. This is dealt with in 2.4 and 2.7, and in clause 5.

Zy or ZNy

Zero-sequence current in the zigzag winding produces an inherent ampere-turn balance between the two halves of the winding on each limb, and provides a low short-circuit impedance.

Three-phase banks of large single-phase units – use of delta connected tertiary windings

In some countries, transformers for high-voltage system interconnection are traditionally made as banks of single-phase units. The cost, mass, and loss of such a bank is larger than for a corresponding three-phase transformer (as long as it can be made). The advantage of the bank concept is the relatively low cost of providing a spare fourth unit as a strategic reserve. It may also be that a corresponding three-phase unit would exceed the transport mass limitation.

The three single-phase transformers provide independent magnetic circuits, representing high magnetizing impedance for a zero-sequence voltage component.

It may be necessary to provide a delta equalizer winding function in the bank, or there may be a need for auxiliary power at relatively low-voltage from a tertiary winding. This can be achieved by external busbar connection from unit to unit in the station. The external connection represents an additional risk of earth fault or short circuit on the combined tertiary winding of the bank.

Different magnetic circuit designs

The most common magnetic circuit design for a three-phase transformer is the three-limb core-form (see figure 1). Three parallel, vertical limbs are connected at the top and bottom by horizontal yokes.

Figure 1 – Three-limb, core-form magnetic circuit

The five-limb, core-form magnetic circuit (see figure 1) has three limbs with windings and two unwound side limbs of lesser cross-section. The yokes connecting all five limbs also have a reduced cross-section in comparison with the wound limbs.
The conventional shell-form three-phase design has a frame with the three wound limbs horizontal and having a common centre line (see figure 3). The core-steel limbs inside the windings have an essentially rectangular cross-section and the adjoining parts of the magnetic circuit surround the windings like a shell.

A new three-phase shell-form magnetic circuit is the seven-limb core, in which the wound limbs are oriented in a different way (see figure 4).

Figure 2 – Five-limb, core-form magnetic circuit

Figure 3 – Three-phase conventional shell-form magnetic circuit

Figure 4 – Three-phase seven-limb shell-form magnetic circuit
The principal difference between the designs, to be discussed here, lies in their behaviour when subjected to an asymmetrical three-phase set of voltages having a non-zero sum i.e. having a zero-sequence component.

This condition may also be described as starting from a zero-sequence current without balancing ampere-turns in any other winding. Such a current appears as a magnetizing current for the magnetic circuit and is controlled by a magnetizing impedance, across which a zero-sequence voltage drop is developed.

The usual types of magnetic circuits behave as follows.

1. Three-limb core-form magnetic circuit

In the three-limb core-form transformer, positive and negative sequence flux components in the wound limbs (which have a zero sum at every instant) cancel out via the yokes, but the residual zero-sequence flux has to find a return path from yoke to yoke outside the excited winding. This external yoke leakage flux sees high reluctance and, for a given amount of flux (a given applied zero-sequence voltage), a considerable magnetomotive force (high magnetizing current) is required. In terms of the electrical circuit, the phenomenon therefore represents a relatively low zero-sequence (magnetizing) impedance. This impedance varies in a non-linear way with the magnitude of the zero-sequence component.

Conversely, uncompensated zero-sequence current constitutes a magnetizing current which is controlled by the zero-sequence magnetizing impedance. The result is a superposed asymmetry of the phase-to-neutral voltages, the zero-sequence voltage component.

The zero-sequence yoke leakage flux induces circulating and eddy currents in the clamping structure and the tank, generating extra stray losses in these components. There could also be increased eddy losses in the windings caused by the abnormal stray flux. There are limitations to the magnitude of any long duration neutral current which is allowable in service. This is considered in 2.4.

2. Five-limb core-form, or shell-form magnetic circuit

In a five-limb core-form, or a shell-form transformer, there are return paths available for the zero-sequence flux through unwound parts of the magnetic circuit (side limbs of five-limb core, outside parts of the shell frame plus, and for the seven-limb shell-form core, the two unwound inter-winding limbs). The zero-sequence flux sees low magnetic reluctance equivalent to a very high magnetizing impedance, similar to that of normal positive-sequence voltage. This applies up to a limit, where the unwound parts of the magnetic circuit reach saturation. Above that, the impedance falls off, resulting in peaked, distorted current.

A three-phase bank of single-phase transformers reacts similarly. The magnetic circuits are separate and independent at any applied service voltage.

Due to the phenomena described above, it is customary to provide such transformers or transformer banks with a delta-connected stabilizing winding (see clause 1).
Characteristic properties and application of auto-connected transformers

3.1 By definition, an auto-connected transformer is a transformer in which at least two windings have a common part (see 3.1.2 of IEC 61726-1).
The single line diagram of an auto-transformer is shown in figure 5. The high-voltage side of the transformer (identified with $U_1$, $I_1$ in the figure) consists of the common winding together with the series winding. The low-voltage side ($U_2$, $I_2$) consists of the common winding alone. The high- and low-voltage systems are electrically connected.

\[
U_1 I_1 = U_2 I_2 = S
\]

\[
\frac{U_1 - U_2}{U_1} = \frac{I_1 - I_2}{I_2} = \alpha
\]

\[
(U_1 - U_2) I_1 = U_2 (I_1 - I_2) = \alpha S
\]

**Figure 5** – Auto-connected transformer, single-line diagram

\(\alpha\) **The reduction factor or auto-factor**

The auto-transformer is physically smaller and has lower losses than a separate winding transformer for the same throughput power. The relative saving is greater the closer the transformation ratio is to unity. The two windings (series and common) represent the same equivalent power ratings or, expressed in other terms, balancing ampere-turns. The relations shown in figure 5 immediately explain the reduction factor, \(\alpha\), of the auto-connection. If \(S\) is the rated power of the auto-connected windings, noted on the rating plate, then the transformer is similar, with regard to physical size and mass, to a separate winding transformer having rated power \(\alpha \times S\). This is often referred to with expressions such as intrinsic rated power or equivalent two-winding rating.

**Example**

An auto-connected transformer \(24/42\ kV, 30\ MVA\), is comparable with a separate winding transformer having a rated power of:

\[
((24 - 42)/42) \times 30 = 12.5\ MVA
\]

If the transformer in addition is provided with a non-auto-connected tertiary winding of \(1\ MVA\) rated power (YNauto \(24/42/1\ MVA\)), then its equivalent two-winding rating will be

\[
(12.5 + 12.5 + 1) = 17.5\ MVA
\]

\(\tau.\) **Short-circuit impedance and leakage flux effects**

The short-circuit impedance of a transformer may be described physically in terms of the reactive power in the leakage field. This in turn depends on the physical size and geometry of the windings.
For an auto-transformer with its reduced dimensions, the reactive power in the leakage field is naturally smaller than for a separate winding transformer with the same rated power. Its impedance, expressed as a percentage, will then be correspondingly lower. The auto-connection factor, $\alpha$, is also a benchmark for the percentage impedance.

However, it may also be observed that if the percentage impedance of an auto-transformer is specified with an elevated value (with a view to limiting fault-current amplitudes in the secondary-side system) then this transformer will, from a design point of view, be a physically small unit with a quite large leakage field. This will be reflected as higher additional losses (winding eddy loss as well as stray field loss in mechanical parts) and possibly even saturation effects due to leakage flux circulating in part through the magnetic circuit. Such effects would restrict the loadability of the unit above rated conditions, but this is not revealed by standard tests.

The transformer loading guide, IEC 45306, takes these phenomena into account when separating between large and medium power transformers. Auto-transformers are to be classified according to their equivalent power rating, and the corresponding percentage impedance, instead of by the rating-plate figures.

7.4 System restrictions, insulation co-ordination

The direct electrical connection between the primary and secondary (three-phase) systems implies that they will have a common neutral point and that the three-phase connection of the auto-transformer is in star. In practice, the systems will normally be effectively earthed and the neutral point of the auto-transformer will usually be specified with reduced insulation level.

- If the transformer neutral is to be directly earthed, the necessary insulation level is very low (see 2.5.1 of IEC 67006-3).

- It may alternatively be foreseen that not all neutrals of several transformers in a station will be directly earthed. This is in order to reduce the prospective earth fault currents. The unearthed neutrals will, however, usually be provided with a surge arrester for protection against transient impulses. The specified arrester rated voltage and the insulation level of the neutral will be co-ordinated with the power frequency voltage appearing at the unearthed neutral during a system earth fault.

- In extra-high-voltage systems with long overhead lines, the possibility of successful single-pole reclosing may be improved by specially tuned reactor earthing. This requires a relatively high insulation of the transformer neutral, which is connected via the tuning reactor to earth.

The series winding of an auto-transformer sometimes presents design difficulties for the insulation across the winding. It is assumed that the X-terminal, the low-voltage side-line terminal, stays at low potential at the incidence of a transient overvoltage on the high-voltage side-line terminal. The stress corresponding to the whole impulse insulation level of the high-voltage side will therefore be distributed along the series winding only. This represents a correspondingly higher turn-to-turn voltage, compared with an overvoltage across the low-voltage side, distributed along the common winding.

7.5 Voltage regulation in system-interconnection autotransformers

Variation of the voltage ratio in an auto-connected transformer may be arranged in different ways. Some of these follow the underlying principles of 1.1 of IEC 67006-1. Others do not because the number of effective turns is changed in both windings simultaneously.
The tapping turns will be either at the neutral terminal or at the joint between the common and the series windings (common point) (see figure 6).

3.5.1 Tapping turns at the neutral

Regulation at the neutral simultaneously increases or decreases the number of turns in both the high-voltage and low-voltage windings but the ratio between the windings changes. This type of regulation will be insufficient in the sense that it requires many regulating turns for the specified range of variation of ratio. Therefore, the volts per turn in the transformer will vary considerably across the tapping range (variable flux). The phenomenon gets more pronounced the closer the ratio of the transformer approaches unity (low \( \alpha \) value). This has to be covered by a corresponding over-dimensioning of the magnetic circuit. It will also result in unequal voltages per step.

The obvious advantage of regulation in the neutral is that the tapping winding and the tap-changer will be close to neutral potential and require only low insulation level to earth.

**Figure 6 – Tapping turns at the common neutral**

3.5.2 Tapping turns at the X-terminal

Regulation arranged at the auto-interconnection in the transformer (the low-voltage side-line terminal) requires the tapping winding and tap-changer to be designed with the insulation level of the X-terminal. They will be directly exposed to steep-front voltage transients from lightning or switching surges. Figure 7 shows a number of different arrangements.
a) The number of turns in the common winding remains unchanged. This is a logical choice if the low-voltage system voltage remains relatively constant while the high-voltage system voltage is more variable.

b) This alternative is the opposite to a). The number of turns facing the high-voltage system voltage remains constant, while the effective number of turns of the low-voltage side varies.

c) The number of turns is constant on the high-voltage side, but for a specific number of reconnected turns, the ratio varies more than in case b). Case b) on the other hand permits plus-minus utilization of the tapping winding by reversing it as indicated in the figure.

Figure 7 – Tapping turns at the lower voltage terminal

1 Zero-sequence properties – neutral load current and earth fault conditions, magnetic saturation and inrush current

This clause outlines the characteristics of three-phase transformers and banks of single-phase transformers with regard to asymmetrical three-phase service conditions.

There are differences depending on the geometry of the magnetic circuit and on the combination of three-phase connections of the windings.

The asymmetrical conditions comprise transient disturbances as well as asymmetries during continuous service, giving rise to:

– temporary loss of symmetry of three-phase voltages and, consequently, of the symmetry of magnetization of the core;
– temporary or permanent asymmetry of load currents, particularly current in the neutral, which will affect the voltage stability, leakage flux and core magnetization.

1.1 Introduction of the symmetrical components of a three-phase system

A short explanation of the conventional analytical method called symmetrical components, which is frequently referred to in power system analysis, is given in 1.1.1. For further information on this method and its application, see textbooks on power system analysis.
A further explanation regarding the practical aspects of earthing of the system through transformer neutrals is given in §4.1.2.

§4.1.1 Principles and terminology of symmetrical components of voltage and current

The method, as conventionally applied, presupposes synchronous and sinusoidal voltages and currents, linked by circuit elements in the form of constant impedance or admittance, with equal value for the three phases. These assumptions imply that all circuit equations are linear, and that changes of variables by linear transformations are possible. One such transformation is that of symmetrical components.

In the general asymmetrical case, the three individual phase voltages or phase current have unequal amplitudes and are not spaced equally in time (not 120 electrical degrees apart). The sum of the momentary values may be different from zero. The phasor picture is an asymmetrical star. The vectorial sum of the three phasors does not necessarily form a closed triangle (non-zero sum).

It is however always possible to replace the original three asymmetrical variables by a combination of the following three symmetrical components:

– a positive sequence component having a fully symmetrical, ordinary set of three-phase voltages or currents;
– a negative sequence component having another symmetrical set, but with opposite phase sequence;
– a zero sequence component having the same phasor value in all three phases with no phase rotation.

The two first components each have zero sum at every instant. The third component represents the residual, non-zero sum of the original variables, with one-third appearing in each phase.

The advantage of the method of symmetrical components for calculation of voltages and currents is that the original system of three coupled equations with three unknown variables is replaced by three separate, single-phase equations with one unknown, one for each component. Each equation makes use of the relevant impedance or admittance parameters for the respective component.

The solution of the equations for the separate symmetrical components are then superposed back, phase by phase, to obtain the phase voltages or currents of the real system.

The algorithms for transformation of the original phase quantities into symmetrical components and back again can be found in appropriate textbooks.

§4.1.2 Practical aspects

The properties of the components have the following practical consequences with regard to currents and voltages.

– The three line currents in a system without earth return or neutral conductor have zero sum. Their transformation into symmetrical components contains positive and negative sequence components but no zero-sequence component.

The currents from a system to a delta-connected winding have this property.
If there is neutral current to earth or through a neutral conductor (fourth wire), then the system of phase currents may have a zero-sequence component. This is a normal condition in four-wire distribution systems with single-phase loads applied between phase and neutral. High-voltage transmission lines do not normally carry any intentional neutral load current. To the extent that load asymmetry exists, it rather has the character of load between two phases which results in a negative-sequence component, but no zero-sequence component.

A zero-sequence component is defined as existing in phase, and with the same amplitude, in all three phases. A zero-sequence component of current is, consequently, precisely one-third of the neutral current.

The set of line-to-line voltages across a delta-connected winding have zero sum, because of the closed connection, and consequently do not contain any zero-sequence voltage component. But inside the delta winding, there may flow zero-sequence current, a short-circuit current circulating around the delta, which is induced from another winding (see § 5.4).

### § 5.4 Impedance parameters for symmetrical components

The impedance (or admittance) parameters of different elements of the system may be different for the three components. In practice, components such as transformers and reactors have equal parameters for positive sequence and negative sequence impedance. For a transformer, they are taken as the values measured during the routine tests.

The zero-sequence parameters of a transformer, however, are different. It may be that transformers having equal values of positive-sequence reactance still have unequal zero-sequence characteristics depending on the type of magnetic circuit, the connection and location of the different winding, the way of guiding leakage flux, etc.

In some cases, a zero-sequence impedance will be non-linear. This is described with reference to the physics of the transformer in the following clauses. They also provide some approximate quantitative estimates for general guidance. If more accurate evidence about a specific transformer is wanted, measurements of its zero-sequence characteristics may be carried out as a special test, on request (see § 7.1 of IEC 6700-1).

### § 5.7 Single-line equivalent diagram of the transformer for zero-sequence phenomena

The fundamentals of the symmetrical component method have been outlined in § 1.1, § 1.4, § 2.1 and § 2.4. It was stated that the analysis of asymmetrical, linear, sinusoidal phenomena is handled in the form of simultaneous, single-phase equations, one for each component. For positive and negative sequence, the transformer is represented with its normal no-load and short-circuit impedances but, for zero-sequence, the diagram is sometimes different, depending on the design. Quantitative information about the zero-sequence parameters can be found in this subclause.
The equivalent diagram of a two-winding three-phase transformer for zero-sequence is composed of a series impedance and a shunt branch. In figure 8, the sum of the two series impedance elements $Z_A$ and $Z_B$ is equal to the ordinary short-circuit impedance for positive-sequence current. The subdivision between the two elements is arbitrary, and either can be put equal to zero.

$Z_m$ is a magnetizing impedance, the order of magnitude of which depends on the design of the magnetic circuit. A five-limb core or a shell-form three-phase magnetic circuit presents very high magnetizing impedance for zero-sequence voltage (see 4.4). A three-limb core, on the other hand, has a moderate magnetizing impedance for zero-sequence voltage. This impedance is non-linear with the current or voltage magnitude and varies from design to design. The yoke leakage flux (see 4.4) induces flow of eddy currents around the whole tank. There is, therefore, a difference between transformers having corrugated tanks of thin steel sheet and those having tanks of flat boilerplate. For boilerplate tank transformers, the per unit zero-sequence impedance is, in general, of the order of 0.1 % to 1.0 % when the neutral current $\pi x I_0$ is equal to the rated current of the winding. The general variation of impedance with current is shown in figure 9.

For a new transformer, the manufacturer will perform a measurement of the zero-sequence impedance on request (see 1.1.3 and 1.3 of IEC 61798-1).
The consequences for particular cases of transformer connections are described in 4.3.4 and 4.3.5.

4.3.4 YNyn transformer without additional delta winding

When both winding neutrals are connected to effectively earthed systems, zero-sequence current may be transferred between the systems, meeting low impedance in the transformer. The system impedances are not, in this case, larger than the transformer series impedance. With a three-limb core, the moderate magnetizing impedance is not negligible. It lowers the effective through impedance of the transformer to approximately 4% to 9% of the positive-sequence short-circuit impedance. With a five-limb core or a shell-form transformer, there is no such reduction.

If the opposing winding system does not accept zero-sequence current, the input impedance of either winding is the magnetizing impedance, which is dependent on the magnetic circuit design as outlined above.

If the opposing winding system has its neutral earthed through an impedance element $Z_n$, this is represented in the zero-sequence diagram by an additional series impedance equal to $\frac{3}{4}Z_n$ (see figure 1.1).

![Diagram](image)

Figure 1.1 – YNyn transformer with neutral earthing impedance – zero-sequence representation

4.3.5 YNyd, or YNyn + d transformer

This is a three-winding combination. There is a star configuration of series impedance elements, in combination with the magnetizing impedance for zero-sequence. In figure 1.1, $Z_A + Z_C$ is the short-circuit impedance between winding A and the delta-connected third winding C, within which a zero-sequence current may circulate (see 4.3). This impedance is the input impedance for zero-sequence current from system I into winding A.

Similarly, the impedance for zero-sequence current from system II into winding B is $Z_B + Z_C$. 

17
Figure 11 – YNynd transformer – zero-sequence representation

The magnetizing impedance $Z_m$ which is also indicated in figure 11, is usually neglected in calculations for this winding combination. It is accepted that the zero-sequence impedances of the diagram differ somewhat from values measured with positive sequence current. The difference depends on the arrangement of the windings with respect to each other and usually stays within 10% to 50%.

Magnetizing impedance under asymmetrical conditions – zero-sequence voltage and magnetic circuit geometry

For several reasons, the symmetry of three-phase voltages in transmission systems under normal service conditions is maintained quite well and does not in general cause any concern for the operation of the transformer.

During asymmetrical earth faults in the network, the system of phase-to-earth voltages contains a zero-sequence component. The degree of asymmetry depends on the method of system earthing. The system is characterized by an earth fault factor which is, briefly, the ratio between phase-to-earth a.c. voltage on an unfaulted phase during the fault and the symmetrical phase-to-earth voltage prior to the fault. This is of importance with regard to insulation coordination.

If the three-phase limbs of a transformer are subjected to a system of induced voltages which contains a zero-sequence component (i.e. has a non-zero sum), then the reaction depends on the magnetic circuit geometry and the connection of the windings.

In a three-limb core type transformer (see figure 11), the unequal flux contributions from the three limbs do not cancel in the yokes. The residual, zero-sequence flux instead completes its path outside the iron core. This represents high reluctance and a low magnetizing impedance for zero-sequence voltage. Quantitative information is given in 3.5. The phenomenon of considerable flux leaving the magnetic circuit and closing outside may also occur during switching transient conditions.
In a five-limb core type transformer (see figure 21), the unwound outer limbs present a low-reluctance return path, where zero-sequence flux may pass. The corresponding magnetizing impedance is high, as for normal positive-sequence flux. The same applies for shell-form three-phase transformers, and, of course, for a bank of three separate single-phase units.

However, applied zero-sequence voltage and current is also influenced by the winding three-phase connection; see the following clauses.

§ 4.0 Zero-sequence and delta windings

The phase-to-phase voltages across a delta-connected winding automatically sum up to zero because of the closed triangle connection. Alternatively, a delta winding can be looked on as a short circuit with regard to zero-sequence voltages.

Zero-sequence current cannot be exchanged between the three terminals of the delta winding and an external system. But a circulating short-circuit current may be induced from another (YN-connected) winding (see figure 31). The zero-sequence impedance of the transformer, seen from the other winding, has the character of a short-circuit impedance between the other winding and the delta winding. For quantitative information, see § 4.7.
Zero-sequence and zigzag windings

In a zigzag connected winding (see figure 41), each limb of the transformer carries part windings from two phases which have opposite winding directions. The number of ampere-turns of a zero-sequence current component cancel out on each limb, with no resulting magnetization. The current meets only a low short-circuit impedance associated with the leakage flux between the part windings on the limb (see also $I_0, Y, Y'$).

Figure 41 – A zigzag connected winding inherently balanced for zero-sequence current

Zero-sequence impedance properties of different transformer connections

Previous subclauses have described zero-sequence characteristics of specific magnetic circuits and of specific individual windings in transformers. This subclause summarizes the zero-sequence characteristics of whole transformers having usual winding combinations.

Table 1 indicates approximate zero-sequence impedance values for two and three-winding combinations when either of the windings is excited from a system. This table as it stands is valid for designs with concentric windings, here numbered $(1)-(3)$ with $(1)$ as the outermost winding. The winding symbols in the first column are written in the same order. It is unimportant which one is the high-voltage winding.

The following subclauses provide further descriptive text.

In table 1, the symbol YN indicates that the winding neutral is earthed directly or via a low impedance. Symbol Y indicates that the neutral is not connected to earth.

Percentage figures, when given, are in relation to the usual reference impedance $U'^{2}/S$.

Some connections are marked with an asterisk (*). In these cases, the zero-sequence current in the excited winding is not balanced by current in any other winding. The zero-sequence impedance is then a magnetizing impedance of relatively high or very high value, depending on the magnetic circuit.

In all other cases, there is current balance between the windings, and the zero-sequence impedance is equal to, or at least close to, the ordinary short-circuit impedance between the windings involved.

The table only indicates the contribution of the transformer itself. Impedances of associated systems are regarded as negligible.
This means that, in the zero-sequence diagram representation, a YN output winding is regarded as having all three phases short-circuited to earth.

Table 1 – Zero-sequence impedances, typical values

<table>
<thead>
<tr>
<th>Winding symbol</th>
<th>Impedance %</th>
<th>Subclauses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Excited winding, τ-limb core</td>
<td>Excited winding, δ-limb core (or shell)</td>
</tr>
<tr>
<td>(') (') ('')</td>
<td>(') (') (')</td>
<td>(') (') (')</td>
</tr>
<tr>
<td>YN Y *</td>
<td>1,1</td>
<td>t,τ, i,i</td>
</tr>
<tr>
<td>Y YN *</td>
<td>1,1</td>
<td>t,τ, i,i</td>
</tr>
<tr>
<td>YN YN D</td>
<td>a.z1, z1, z1</td>
<td>τr, τr, τr</td>
</tr>
<tr>
<td>D YN</td>
<td>a.z1, z1, z1</td>
<td>τr, τr, τr</td>
</tr>
<tr>
<td>YN Y Y*</td>
<td>1,1</td>
<td>t,τ, i,i</td>
</tr>
<tr>
<td>Y YN Y*</td>
<td>1,1</td>
<td>t,τ, i,i</td>
</tr>
<tr>
<td>Y Y YN*</td>
<td>1,1</td>
<td>t,τ, i,i</td>
</tr>
<tr>
<td>YN YN Y</td>
<td>a.z1, z1, z1</td>
<td>τr, τr, τr</td>
</tr>
<tr>
<td>YN Y YN</td>
<td>a.z1, z1, z1</td>
<td>τr, τr, τr</td>
</tr>
<tr>
<td>Y Y YN YN</td>
<td>a.z1, z1, z1</td>
<td>τr, τr, τr</td>
</tr>
<tr>
<td>YN YN D</td>
<td>a.(Z1 + Z1)</td>
<td>a.τr + a.τr</td>
</tr>
<tr>
<td>YN D D</td>
<td>a.τr + a.τr</td>
<td>τr, τr, τr</td>
</tr>
<tr>
<td>YN Y D</td>
<td>a.τr + a.τr</td>
<td>τr, τr, τr</td>
</tr>
<tr>
<td>D YN YN</td>
<td>a.τr + a.τr</td>
<td>τr, τr, τr</td>
</tr>
<tr>
<td>D YN Y</td>
<td>a.τr + a.τr</td>
<td>τr, τr, τr</td>
</tr>
<tr>
<td>D YN D</td>
<td>a.τr + a.τr</td>
<td>τr, τr, τr</td>
</tr>
</tbody>
</table>

NOTES

1. z1, τr, and Zr are short-circuit positive-sequence impedances.

\[
z_1 = \frac{Z_{11} + Z_{1τ} - Z_{ττ}}{τr}, \text{ similarly } Z_r \text{ and } Z_τ.
\]

2. a1, a2, and a3 are multiplying factors generally in the range 1 < a1 < a2 < a3 < 1

3. Particular aspects of zero-sequence impedance properties are given in 1.7, 2.7, 3.7, and 4.7

4. Connections marked with an asterisk (*) indicate cases where the zero-sequence impedance is a magnetizing impedance of relatively high or very high value, depending on the nature of the magnetic circuit.
The transformer receives and transfers zero-sequence current between the two systems, provided that the neutrals are earthed. It then presents normal short-circuit impedance for the current.

If the neutral of an auto-connected transformer is not earthed, transfer of zero-sequence current is still possible but it meets a different impedance.

If no transfer of incoming zero-sequence current from a system to the opposing system is possible, then the transformer presents magnetizing impedance to the current. This magnetizing impedance is very high in five-limb core-form transformers, in shell-form transformers, and also in a bank of three single-phase transformers.

\[ YNyn \text{ or } YNauto \text{ without delta winding} \]

The transformer presents low impedance (of short-circuit impedance character) to zero-sequence current from an effectively earthed system into a yn-connected winding. A circulating current around the delta winding provides compensating ampere-turns (see figure 51).

\[ YNd \text{ or } Dyn \text{ or } YNynd \text{ or } YNyn + d \text{ (equalizer winding)} \]

The transformer presents low impedance (of short-circuit impedance character) to zero-sequence current from the Z side system. There is inherent ampere-turn balance for the zero-sequence current within the Z-winding itself.

This is why an additional delta-connected equalizer winding in a Yy transformer (or bank of three single-phase transformers) serves to reduce the zero-sequence impedance of the connected system and thereby its earth fault factor (see 4.4). A consequence is that the prospective earth fault current increases.

It is important to ensure that the short-circuit withstand strength of a tertiary delta winding or an equalizer winding is sufficient for the maximum induced zero-sequence current during an earth fault in either of the connected systems. Alternatively, built-in current-limiting reactors may be connected inside the delta in order to bring down the induced fault current to a tolerable value.

\[ Yzn \text{ or } ZNy \]

The transformer presents low impedance (of short-circuit impedance character) to zero-sequence current from the Z side system. There is inherent ampere-turn balance for the zero-sequence current within the Z-winding itself.
This is why a transformer with a ZN-connected winding is used to provide a neutral point for connection of a neutral earthing impedance to a system when the winding of the main transformer is a delta winding. The Z-connected transformer is referred to as an earthing transformer or neutral coupler; see section 6 of IEC 67006-1. The same function can be achieved with a winding combination YNd on the earthing transformer.

If the Y side has its neutral connected to earth (YNzn), the transformer presents magnetizing impedance to zero-sequence from that side, about the same as YNyn above. The Z-winding, which is inherently balanced for zero-sequence, cannot provide compensating ampere-turns for zero-sequence current in the opposing Y-winding.

4.4 Continuous zero-sequence loading (neutral point current)

A single-phase load on a three-phase transformer may be placed either between phases or between phase and neutral.

In the first case, the system of currents on the primary and secondary sides contain positive and negative sequence components, but not zero-sequence. The distribution of phase currents on the primary side of the transformer depends on the three-phase connection. It is not possible to convert a single-phase load to a symmetrical three-phase load on the primary side by a particular transformer connection. The allowable loading is referred to rated current of the respective windings.

If the load is drawn between phase and neutral of the transformer, there may be other restrictions than those given by the rated current of the winding. Subclause 8.1 of IEC 67006-1 requires that the neutral terminal shall always be dimensioned for the recognised earth fault current and for continuous load current through the neutral when specified (this is a normal condition for distribution transformers). It is required in annex A of IEC 67006-1 that an enquiry shall contain information on:

- the intended method of operation of the system to which the transformer windings will be connected, particularly when an equalizer winding is specified;
- any anticipated unbalanced loading.

In line with what has been described in the previous subclauses, a secondary winding neutral may in general be continuously loaded as follows, depending on winding connections, magnetic circuit design, and system earthing:

- a Dyn transformer neutral may be loaded with rated current of the winding;
- a YNyn transformer with both neutrals earthed may be loaded with rated current through its neutrals, provided that the system earthing permits this (with regard to voltage asymmetry);
- a Z-connected winding neutral may be loaded with rated current;
- a Yyn + d transformer (a transformer provided with an equalizer winding) may have its secondary neutral terminal loaded with current up to the rated current, provided that the delta-connected winding has a power rating equal to at least one-third of the power rating of the secondary winding. (The circulating current per phase of the tertiary winding balances the zero-sequence current in the secondary winding, which is, by definition, one-third of the current in the neutral);
– in a Yynd transformer which has a loadable tertiary, this tertiary will function in the same way as an equalizer winding (see previous indent). Any circulating current in the delta winding will combine with the external load current of that winding. (The total current may be measured if there are current transformers inside the delta connection in the transformer);

– a Yyn transformer without any additional delta-connected winding does not provide well-defined symmetry of the phase voltages. (This case presupposes that the primary neutral is not earthed.)

– a three-limb, core-form distribution transformer having a Yyn connection is not in general suitable for loading between phase and neutral. The voltage asymmetry will be objectionable if the neutral carries more than about $1\%$ of the rated current of the winding. Therefore, mixed connections, Dyn or Yzn, are preferred for distribution transformers feeding four-wire distribution systems;

– a medium-size, medium voltage transformer with this connection may carry about $5\%$ of the rated current for a duration of $2$ h to an arc-suppression coil, in addition to full symmetrical load, without thermal risk. Such requirements should, however be specifically confirmed.

\[4.9\]

*Transister reluctance and magnetizing impedance, steady-state saturation under abnormally high power frequency voltage*

An equivalent single-line diagram of the power transformer contains a shunt element, representing the excitation current of the magnetic circuit. In normal service, this current is very small and negligible, e.g. in conjunction with voltage drop calculations, which justifies the pattern of the equivalent circuit (see clause 7). In other words, the magnetizing inductance is very high. In terms of magnetization characteristics, the reluctance which the magnetic flux encounters is very low, i.e. the path of the main flux is easily magnetized.

If a power frequency voltage applied across the windings of any limb of the transformer is abnormally high, the core material gets saturated during part of each half-cycle. During the saturated condition, the magnetic reluctance increases considerably. A peaked, drastically increased magnetizing current is drawn from the power source.

During phenomena with saturation of the core, there is also a considerable flux outside the core-steel, between the core and the winding. This may induce high eddy currents in metallic parts outside the windings, resulting in local heating and discharges across unintentional contacts.

A phenomenon which may give rise to local saturation in a transformer is the excessive magnetic leakage flux occurring during the flow of heavy overload current. The leakage flux passes between the windings and part of it has a return path through the magnetic circuit. In addition, it is likely that, under these conditions, the service voltage is also abnormal. The combined result may be unforeseen saturation conditions in certain parts of the core.

\[4.10\]

*Transient saturation, inrush current*

When a transformer is suddenly energized with full system voltage, a random saturation phenomenon may occur, which is usually referred to as an inrush current (see figure 1). In steady state, the volt-time integral of a full half-cycle of unidirectional voltage between two zero passages, applied across a phase of the winding, corresponds to the flux swing from full density in one direction to full density in the other direction.
Immediately upon energization, however, a disturbed transient condition occurs. Depending on the direction of the existing remanence in the magnetic circuit and the point-of-wave at which the voltage is applied, the transient flux density may reach the saturation limit of the core-steel and have to rise above that value, before the applied voltage changes sign. The transient magnetizing current amplitude may reach any peak value up to a maximum which may be higher than the rated current and approach the through-fault short-circuit current of the transformer.

![Image](image-url)

**Figure 6.1 – Inrush transients**

From the description of the phenomenon, it is evident that it is a random occurrence which develops in full only now and then in several energizations. The inrush current from the system appears with different magnitude in different phases. When a delta-connected winding or a star-connected winding with effectively earthed neutral is energized, the phenomenon is linked to the individual limb, while in a star winding without neutral current connection, a combination of two-limb windings in series is initially involved.

When high inrush current occurs, it is always offset and appears with high amplitudes of one polarity only. The inrush current therefore contains a d.c. component which decays in less than 1 s. The decay time is longer for transformers having low-loss core material and it tends to be longer for large transformers. This d.c. component and the high harmonic content of the current
are of importance to relay protection circuits. It may also cause saturation effects in an already energized transformer with which the switched transformer is connected in parallel. The phenomenon is accompanied by a considerable thud sound, and it takes many seconds, or even minutes, before the humming of the transformer returns to normal.

The prospective extreme value of inrush current, expressed as a multiple of rated current, depends on the selected service flux density of the transformer design. Higher values are encountered with present-day core materials than in the past. The configuration of the windings is also important e.g. whether an inner or outer winding of a concentric pair is energized. An outer winding has higher air-core inductance and will draw lower inrush current from the system.

\[ \text{Geomagnetically induced current and parasitic currents from d.c. systems} \]

A high-voltage a.c. system which is effectively earthed through its transformer neutrals provides a low-resistance path for d.c. or quasi-d.c. current flowing in the crust of the earth.

Geomagnetically induced currents are encountered mostly in temperate zones of the earth with high-resistivity gravel soil. They appear as slowly varying pulses (several minutes) with a magnitude of tens of amperes in transformer neutrals.

Parasitic currents are earth-return currents from d.c. traction systems, cathodic protection systems etc. They may reach a magnitude of a few amperes in the neutral.

When a transformer is subjected to such d.c. current in the neutral, this results in a d.c. magnetization bias of the magnetic circuit. The magnetization current becomes strongly asymmetrical to a degree which will compensate the impressed d.c. current. It also has a high harmonic content. These d.c. currents have several consequences.

- The transformer sound level will increase significantly.
- The current harmonics may cause relay malfunction and false tripping.
- The harmonics may even cause considerable stray flux overheating.

The severity of the phenomena is dependent on the ability of the d.c. current to magnetize the core and also on the core design.

\[ \text{Calculation of short-circuit currents in three-winding, three-phase transformers} \]

\[ \text{(separate winding transformers and auto-connected transformers) with earthed neutrals} \]

\[ \text{General} \]

Short-circuit current calculations in three-phase a.c. systems are described in IEC 909, IEC 1094-1 and IEC 1094-2.

Clause 9 provides equations for the calculation of currents flowing through the different windings and terminals of the transformer during system faults of different types.

The connection of the transformer is YNyn d or YNauto d (or YNyn + d or YNauto + d if the third winding is a non-loadable stabilizing winding).
Notations of systems and windings

The three windings and their associated systems are referred to with roman numerals.

I is the high-voltage winding or system.
II is the intermediate voltage winding or system.
III is the tertiary or stabilizing winding.

Windings I and II are Y-connected with earthed neutrals.

The three line terminals of each winding are referred to with the capital letters A, B and C.

A phase winding in the Y-connected winding is identified with the same letter as its line terminal. A phase winding in the delta-connected tertiary is identified with two letters, AB, BC and CA.

Symmetrical components of voltage, current, or impedance are identified with indices +, −, 0 in superscript position, for example:

\[ Z_{S1}^+ \] the positive-sequence impedance of system I;
\[ U_{II}^0 \] the zero-sequence voltage of system II.

The complex three-phase phase-shift operator is

\[ \alpha = -\frac{1}{\sqrt{3}} + j\frac{\sqrt{3}}{\sqrt{3}} \] \[ = e^{j\frac{\pi}{2}} \]

\[ \alpha' = -\frac{1}{\sqrt{3}} - j\frac{\sqrt{3}}{\sqrt{3}} \] \[ = e^{-j\frac{\pi}{2}} \]

Transformer parameters

Reference power for percentage notations:

\[ S_r \] (the rated power of the main windings I and II).

Reference voltages for the windings:

\[ U_I, U_{II}, U_{III} \] (the rated voltages of the windings).

Reference currents of the windings:

\[ I_I, I_{II}, I_{III} \] (the rated currents of the windings).

The reference impedances of the windings, consequently:

\[ Z_{r(I)} = \frac{U_I^*}{S_r}; \quad Z_{r(II)} = \frac{U_{II}^*}{S_r} \]

The index within the brackets indicates the voltage system to which the impedance is referred.
Definition of per unit or percentage values of impedance:

\[z_{I,II} = \frac{Z_{I,II(I)}}{Z_{r(I)}} = Z_{I,II} = \frac{Z_{I,II(II)}}{Z_{r(II)}} = Z_{I,II} = \frac{Z_{I,II(III)}}{Z_{r(III)}} = \frac{S_r}{U_I} \]

where

- \(Z_{I,II(I)}\) is the impedance in ohms per phase between windings I and II referred to voltage I;
- \(Z_{I,II(II)}\) is the impedance in ohms per phase between windings I and II referred to voltage II;
- \(Z_{I,II}\) is the impedance in per unit (or percentage) between windings I and II. This expression depends on the power rating \(S_r\) and is independent of the voltage side.

All per unit or percentage expressions of voltages, currents or impedances are identified with lower-case letters instead of capitals.

The transformation of the three-winding system into a star equivalent network and the calculation of the branch impedances, expressed in per unit form are as follows:

\[z_I = \frac{1}{2}(z_{I,II} + z_{I,III} - z_{II,III})\]
\[z_{II} = \frac{1}{2}(z_{II,III} + z_{I,II} - z_{I,III})\]
\[z_{III} = \frac{1}{2}(z_{I,III} + z_{II,III} - z_{I,II})\]

Symmetrical component impedances are as follows.

The positive-sequence impedances are by definition identical to the conventional transformer impedances for symmetrical three-phase current.

The negative-sequence impedances of the transformer are equal to the positive-sequence impedance.

The zero-sequence short-circuit impedance between the two main windings normally differs marginally from the conventional impedance. The difference is of the order of \(\%\) to \(\%\) of the conventional impedance, up or down, depending on winding arrangements. If, however, additional zero-sequence impedance is installed by adding built-in reactors to the delta winding, the zero-sequence impedance will be much higher.

\[Z_{ss} = Z_{ss}^+ = Z_{ss}^- = Z_{ss}^{IK} = \frac{S_r}{U_I} = \frac{S_r}{U_{III}}\]

where \(1 \leq k \leq \%\) (effectively earthed).

Impedances of systems I and II

The system impedances are denoted with index \(s\) in order to distinguish them from transformer short-circuit impedances. They are short-circuit impedances, as seen from the transformer.

Positive and negative-sequence impedances are assumed to be equal, but the zero-sequence impedances are higher:

\[Z_{sl}^+ = Z_{sl}^-\]
\[Z_{sl}^+ = kZ_{sl}^+\]
Analogous relations apply for system II.

Tertiary system impedance does not intervene in any of the calculations presented below.

The transformer neutrals of windings I and II, or the common auto-connection neutrals, are connected to station earth without any additional impedance which would otherwise make an addition to the zero-sequence impedance.

Summary of cases studied in this subclause

Case 1: Single-phase earth fault on system II (figure 71a)
Case 2: Single-phase earth fault on system I (figure 81a)
Case 3: Two-phase earth fault on system II (figure 91a)
Case 4: Two-phase earth fault on system I (figure 10a)
Case 5: Three-phase short circuit on terminal II
Case 6: Three-phase short circuit on terminal I
Case 7: Three-phase short circuit on winding III.

For cases 1 to 4, additional figures (11b to 10b) have been included to show the flow of currents in a three-phase diagram of the transformer with the associated system lines. The figures show auto-transformer connections but the calculation is also valid for separate winding connections.

These additional figures (11b to 10b) also indicate the equivalent single-line impedance network which corresponds to the calculation of short-circuit current by the symmetrical component method. This impedance network contains three blocks, for the positive, negative and zero-sequence impedance elements, respectively.
Figure 7.1a – Case 1: Single-phase earth fault on system II

$$Z^+ = \frac{(Z_{III} + Z_{III}^-)Z_{III}}{Z_{IJ} + Z_{III}^- + Z_{III}}$$

$$Z^- = \frac{(Z_{II} + Z_{III}^-)Z_{III}}{Z_{IJ} + Z_{III}^- + Z_{III}}$$

$$Z'^- = \frac{(Z_{II}^- + Z_{III}^-)Z_{III}}{Z_{II} + Z^- + Z_{III}^-}$$

with $$Z' = \frac{(Z_{I} + Z_{III}^-)Z_{III}}{Z_{II} + Z' + Z_{III}^-}$$

Figure 7.1b – Case 1: Three-phase circuit diagram and symmetrical component impedance network
All impedances are referred to system II.

For this case, see annex A, case 1:

\[ I_\text{II}^+ = I_\text{II}^- = I_\text{II} = I = \frac{1}{\sqrt{3}} \times \frac{U_{\text{II}}}{Z} \]  

(1)

where \[ Z = Z^+ + Z^- + Z^- = \gamma Z^+ + Z^- \]  (because \( Z^+ = Z^- \))  

(2)

The current in the fault is

\[ I_\text{II}^* = \frac{\sqrt{3} \times U_{\text{II}}}{Z} \]  

(3)

Branch currents

From phase A of the system II (faulted phase):

\[ I_{\text{IIA}} = I_{\text{II}}^+ + I_{\text{II}}^- + I_{\text{II}}^* = \gamma I_{\text{II}}^+ + I_{\text{II}}^* = \gamma \left( \frac{Z^+}{Z_{\text{II}}} \times I + \frac{Z^-}{Z_{\text{II}}} \times I \right) \]  

(4)

Transformer winding II, phase A:

\[ I_{\text{IIA}} = \gamma (I^+ - I_{\text{II}}^*) + I^* - I_{\text{II}}^* = \gamma I - I_{\text{IIA}} \]  

(5)

Phase B (unfaulted phase):

\[ I_{\text{IB}} = I_{\text{II}}^* - I_{\text{II}}^* \]  

(6)

Component currents in winding I:

\[ I_i = I_i^* = \frac{U_{\text{II}}}{U_i} \times \frac{U_{\text{II}}}{U_i} \times \frac{U_{\text{II}}}{U_i} (I^* - I_{\text{II}}^*) = \frac{U_{\text{II}}}{U_i} \times I^* - \frac{Z^+}{Z_{\text{II}}} \]  

(7)

\[ I_i = \frac{Z_{\text{II}}}{Z_i^+ + Z_{\text{II}}^+ + Z_{\text{II}}^+} \times \frac{U_{\text{II}}}{U_i} \times I_{\text{II}} \]  

(8)

where \[ I_i^* = (I^* - I_{\text{II}}^*) = I^* - \frac{Z^-}{Z_{\text{II}}} \]  

Winding I, phase A:

\[ I_{\text{IA}} = \gamma I_i^* + I_i^* \]  

(9)

Other phase:

\[ I_{\text{IB}} = I_i^* - I_i^* \]  

(10)
Current circulating in delta winding:

\[ I_{III} = \frac{U_{III}}{U_{III} \times \sqrt{3}} \times \frac{Z_{I}' + Z_{sl}'}{Z_{I}' + Z_{sl}'} \times (1 - \frac{Z_{III}'}{Z_{III}'}) \]  

(11)

**NOTE** – This case is analogous with case ١. All quantities could be obtained from case ١ just by switching indices I and II.

**Figure ٨١أ – Case ١**: Single-phase earth fault on system I

**Figure ٨١ب – Case ١**: Three-phase circuit diagram and symmetrical component impedance network

**Figure ٨١ – Case ١**
All impedances are referred to system I.

For this case, see annex A, case 1:

\[ I_1^* = I_1^- = I - \frac{U_1}{\sqrt{Z}} \]  

(1.1)

where \( Z = Z^+ + Z^- + Z^' = Z^+ + Z^' \) (because \( Z^+ = Z^- \))  

(1.3)

The current in the fault is

\[ I_1^* = \frac{\sqrt{r} \times U_1}{Z} \]  

(1.4)

Branch currents

From phase A of system I (faulted phase):

\[ I_{slA} = I_{slA}^+ + I_{slA}^- = \gamma I_{slA}^+ + I_{slA}^- = \gamma \frac{Z^+}{Z_{sl}^+} \times I + \frac{Z}{Z_{sl}^+} \times I \]  

(1.5)

Transformer winding I, phase A:

\[ I_A = \gamma (I - I_{slA}) = \gamma (I^+ - I_{slA}^+) + I^- - I_{slA}^- \]  

(1.6)

Phase B (unfaulted phase):

\[ I_B = I_{slB} - I_{slB}^- \]  

(1.7)

Component currents in winding II:

\[ I_{II}^+ = I_{II}^- = \frac{U_1}{U_{II}} \times I^+ = \frac{U_1}{U_{II}} \times I \left( 1 - \frac{Z^+}{Z_{sl}^+} \right) \]  

(1.8)

\[ I_{II}^- = \frac{Z_{III}^1}{Z_{III}^+ + Z_{III}^-} \times \frac{U_1}{U_{II}} \times I \left( 1 - \frac{Z^+}{Z_{sl}^+} \right) \]  

(1.9)

Phase currents:

\[ I_{IIA} = \gamma I_{II}^+ + I_{II}^- \]  

(1.10)

\[ I_{IIB} = I_{II}^- - I_{II}^+ \]  

(1.11)
Current circulating in delta winding:

\[
l_{III} = \frac{U_I}{U_{III} \times \sqrt{3}} \times \frac{Z_{II}^* + Z_{sII}^*}{Z_{II}^* + Z_{sII}^* + Z_{III}^*} \times I'(1 - \frac{Z_{sII}^*}{Z_{sII}^*}) (\text{??})
\]

**Figure 91a – Case III**: Two-phase earth fault on system II

**Figure 91b – Case IV**: Three-phase circuit diagram and symmetrical component impedance network

\[
Z^+ = \frac{(Z_{II} + Z_{sII}^*)Z_{sII}^*}{Z_{II} + Z_{sII}^* + Z_{sII}^*}
\]

\[
Z^- = \frac{(Z_{II} + Z_{sII}^*)Z_{sII}^-}{Z_{II} + Z_{sII}^- + Z_{sII}^-}
\]

\[
Z' = \frac{(Z_{II} + Z_{sII})Z_{sII}'}{Z_{II} + Z_{sII} + Z_{sII}'}
\]

\[
\text{with } Z' = \frac{(Z_{II} + Z_{sII}^*)Z_{sII}'}{Z_{II} + Z_{sII} + Z_{sII}'}
\]
All impedances are referred to system II.

For this case (see annex A, case 2), the components of voltage are

\[ V_{II}^+ = V_{II}^* = V_{II}^- = \frac{U_{II}}{r} \times \frac{Z}{Z^+} \]  

(\^)\(\text{r}\)

where \( \frac{1}{Z} = \frac{1}{Z^+} + \frac{1}{Z^-} + \frac{1}{Z^*} = \frac{Z^+ + Z^-}{Z^+ \times Z^-} \) (because \( Z^+ = Z^- \))  

(\^)\(\text{s}\)

Thus \( V_{II}^* = \frac{Z^*}{Z^+ + Z^-} \times \frac{U_{II}}{r} \)  

(\^)\(\text{o}\)

Components of fault current:

\[ I^+ \times Z^+ = \frac{U_{II}}{\sqrt{r}} - V_{II}^+ = \frac{U_{II}}{\sqrt{r}} \left( - \frac{Z^*}{Z^+ + Z^-} \right) \]

\[ I^* = \frac{U_{II}}{\sqrt{r}} \times \frac{Z^* + Z^-}{Z^+ (Z^+ + Z^-)} \]

\[ I^- = \frac{U_{II}}{\sqrt{r}} \times \frac{-Z^*}{Z^- (Z^+ + Z^-)} \]

\[ I^- = \frac{V_{II}^*}{Z^*} \times \frac{-Z^*}{Z^+ + Z^-} \]  

(\^)\(\text{v}\)

Component currents in system II:

\[ I_{sII}^+ = I^+ \times \frac{Z^+}{Z_{sII}^+} \]

\[ I_{sII}^- = I^- \times \frac{Z^-}{Z_{sII}^-} \]

\[ I_{sII}^* = I^* \times \frac{Z^*}{Z_{sII}^*} \]  

(\^)\(\text{w}\)

Phase currents in system II:

\[ I_{sII A} = I_{sII}^+ + I_{sII}^* + I_{sII}^- \]

\[ I_{sII B} = a^\gamma I_{sII}^+ + a I_{sII}^* + I_{sII}^- \]  

(\^)\(\text{x}\)

\[ I_{sII C} = a I_{sII}^+ + a^\gamma I_{sII}^* + I_{sII}^- \]
Component currents in transformer winding II:

\[
\begin{align*}
    i_{il}^+ &= i^+ - i_{sll}^+ = i^+ (1 - \frac{Z^+}{Z_{sll}^+}) \\
    i_{il}^- &= i^- - i_{sll}^- = i^- (1 - \frac{Z^-}{Z_{sll}^-}) = i^- (1 - \frac{Z^+}{Z_{sll}^+}) \\
    i_{il}^- &= i^- - i_{sll}^- = i^- (1 - \frac{Z^-}{Z_{sll}^-})
\end{align*}
\]  

Currents in the phases of winding II:

\[
\begin{align*}
    i_{IIA} &= i_{ll}^+ + i_{ll}^- + i_{il}^+ \\
    i_{IIB} &= \alpha i_{ll}^+ + \alpha i_{il}^- + i_{il}^- \\
    i_{IIC} &= \alpha i_{ll}^+ + \alpha i_{il}^- + i_{il}^-
\end{align*}
\]  

Component currents in line and winding I:

\[
\begin{align*}
    i_{i}^+ &= \frac{u_{ll}}{u_{i}} \times i_{ll}^+ \\
    i_{i}^- &= \frac{u_{ll}}{u_{i}} \times i_{ll}^- \\
    i_{i}^- &= \frac{u_{ll}}{u_{i}} \times \frac{Z_{ll}^+}{Z_{ll}^+ + Z_{ll}^- + Z_{sll}^-} \times i_{ll}^+
\end{align*}
\]  

Phase currents:

\[
\begin{align*}
    i_{IA} &= i_{i}^+ + i_{i}^- + i_i \\
    i_{IB} &= \alpha i_{i}^+ + \alpha i_{i}^- + i_i \\
    i_{IC} &= \alpha i_{i}^+ + \alpha i_{i}^- + i_i
\end{align*}
\]  

Current circulating in delta winding:

\[
\begin{align*}
    i_{III} &= \frac{u_{ll}}{u_{III}} \times \frac{Z_{i}^- + Z_{sll}^-}{Z_{i}^- + Z_{sll}^- + Z_{III}^-} \times i_i
\end{align*}
\]
NOTE – This case is analogous with case $\gamma$. All quantities could be obtained from case $\gamma$ just by switching indices I and II.

**Figure 1·a – Case 1**: Two-phase earth fault on system I

\[
Z^+ = \frac{(Z_{II} + Z_{III})Z_{sI}}{Z_{II} + Z_{sII} + Z_{sII}^+} \\
Z^- = \frac{(Z_{II} + Z_{III})Z_{sI}}{Z_{II} + Z_{sII} + Z_{sII}^-} \\
Z' = \frac{(Z_i' + Z_i')Z_{sI}}{Z_i' + Z_i' + Z_{sI}'} \\
\text{with } Z' = \frac{(Z_i' + Z_i')Z_{II}}{Z_i' + Z_{sII} + Z_{III}'}
\]

**Figure 1·b – Case 1**: Three-phase circuit diagram and symmetrical component impedance network

**Figure 1·c – Case 1**

All impedances are referred to system I.
For this case, the components of voltage are

\[
V_+^+ = V_-^- = V_0^- = \frac{V_0}{\sqrt{r}} \times \frac{Z}{Z^+} \quad (\text{vii})
\]

where \( Z = \frac{1}{Z^+} + \frac{1}{Z^-} + \frac{1}{Z^0} = \frac{Z^+ + Y Z^0}{Z^+ \times Z^0} \) (because \( Z^+ = Z^- \)) \quad (\text{viii})

Thus

\[
V_+^+ = \frac{Z^-}{Z^+ + Y Z^0} \times \frac{U_0}{\sqrt{r}} \quad (\text{vii})
\]

Components of fault current:

\[
I^+ \times Z^+ = \frac{U_0}{\sqrt{r}} - V_0^+ = \frac{U_0}{\sqrt{r}} (1 - \frac{Z^0}{Z^+ + Y Z^0})
\]

\[
I^+ = \frac{U_0}{\sqrt{r}} \times \frac{Z^+ + Z^0}{Z^+ (Z^+ + Y Z^0)}
\]

\[
I^- = \frac{V_0^-}{Z^0} = \frac{U_0}{\sqrt{r}} \times \frac{-Z^0}{Z^+ (Z^+ + Y Z^0)}
\]

\[
I^- = -\frac{V_0^-}{Z^0} = \frac{U_0}{\sqrt{r}} \times \frac{-1}{Z^+ + Y Z^0} \quad (\text{vii})
\]

Component currents in system I:

\[
I^+_{sl} = I^+ \times Z^0_{sl} \quad (\text{viii})
\]

\[
I^-_{sl} = I^- \times Z^0_{sl} = I^- \times Z^+_{sl} \quad (\text{vii})
\]

\[
I^+_{sl} = I^- \times Z^0_{sl} \quad (\text{vii})
\]

Phase currents in system I:

\[
I_{slA} = I^+_{sl} + I^-_{sl} + I^0_{sl} \quad (\text{viii})
\]

\[
I_{slB} = a^\gamma I^+_{sl} + a I^-_{sl} + I^0_{sl} \quad (\text{viii})
\]

\[
I_{slC} = a^\gamma I^+_{sl} + a I^-_{sl} + I^0_{sl} \quad (\text{viii})
\]
Component currents in transformer winding I:

\[
I_{i}^{*} = I_{i}^{+} - I_{si}^{+} = I_{i}^{+} (1 - \frac{Z_{si}^{+}}{Z_{si}^{*}}) \\
I_{i}^{-} = I_{i}^{-} - I_{si}^{-} = I_{i}^{-} (1 - \frac{Z_{si}^{-}}{Z_{si}^{*}}) \\
I_{i}^{'} = I_{i}^{'} - I_{si}^{'} = I_{i}^{'} (1 - \frac{Z_{si}^{'}}{Z_{si}^{*}})
\]

Currents in the phase of winding I:

\[
I_{IA} = I_{i}^{+} + I_{i}^{-} + I_{i}^{'} \\
I_{IB} = \alpha I_{i}^{+} + \alpha I_{i}^{-} + I_{i}^{'} \\
I_{IC} = \alpha I_{i}^{+} + \alpha I_{i}^{-} + I_{i}^{'}
\]

Component currents in line and winding II:

\[
I_{i}^{*} = \frac{U_{i}}{U_{ii}} \times I_{i} \\
I_{ii} = \frac{U_{i}}{U_{ii}} \times I_{i} \\
I_{ii}^{'} = \frac{U_{i} Z_{iii}}{Z_{ii}^{*} + Z_{ii}^{*} + Z_{si}^{*}} \times I_{i}^{'}
\]

Phase currents:

\[
I_{iA} = I_{i}^{+} + I_{ii} + I_{ii}^{'} \\
I_{iiB} = \alpha I_{i}^{+} + \alpha I_{ii} + I_{ii}^{'} \\
I_{iiC} = \alpha I_{ii} + \alpha I_{ii} + I_{ii}^{'}
\]

Current circulating in delta winding:

\[
I_{iii} = \frac{U_{i} Z_{ii}^{*} + Z_{si}^{*} Z_{ii}^{*}}{U_{iii} \sqrt{3} Z_{ii}^{*} + Z_{ii}^{*} + Z_{si}^{*} + Z_{iii}^{*}} \times I_{i}^{'}
\]
Case Ⅴ: Three-phase short-circuit on terminals II

This case contains positive sequence only.

Fault current per phase on system II:

\[
I_{II} = \frac{U_{II}}{\sqrt{3} \times Z^+}
\]  

(Ⅴ.0)

With \( Z^+ = \frac{Z_{sl}^+ + Z_{l, II}}{Z_{sl}^+ + Z_{l}^+ + Z_{l, II}^+} \) Impedances are referred to system II.

Current per phase in winding I:

\[
I_i = I_{II} \times \frac{U_{II}}{U_l}
\]  

(Ⅴ.1)

There is no circulating current in winding III.

Case Ⅵ: Three-phase short circuit on terminals I

This case contains positive sequence only.

Fault current per phase:

\[
I_i = \frac{U_i}{\sqrt{3} \times Z^+}
\]  

(Ⅵ.1)

With \( Z^+ = \frac{Z_{sl}^+ + Z_{l, II}}{Z_{sl}^+ + Z_{l}^+ + Z_{l, II}^+} \) Impedances are referred to system I.

Current per phase in winding II:

\[
I_{II} = I_i \times \frac{U_l}{U_{II}}
\]  

(Ⅵ.2)

There is no circulating current in winding III.

Case Ⅶ: Three-phase short circuit on winding III

This case contains positive sequence only:

\[
Z^+ = \frac{Z_{III}^+ + (Z_{l}^+ + Z_{l, II})(Z_{sl}^+ + Z_{l}^+)}{Z_{sl}^+ + Z_{l}^+ + Z_{l, II}^+}
\]  

(Ⅶ.1)

Impedances are referred to winding III.
Short-circuit current – line current:

\[ I_{III} = \frac{U_{III}}{\sqrt{3} \times Z^*} \]  

(5.0)

Short-circuit current per phase winding:

\[ \frac{1}{\sqrt{3}} \times I_{III} \]  

(5.1)

Fault contribution from systems I and II:

\[ h_{III} = h_{III} \times \frac{Z_{sI}^* + Z_{II}^*}{Z_{sI}^* + Z_{sII}^* + Z_{sIII}^*} \]  

(5.2)

\[ h_{III} = h_{III} - h_{III} \]  

(5.3)

Currents in windings I and II:

\[ I = h_{III} \times \frac{U_{III}}{U_I} \]  

(5.4)

\[ I_{II} = h_{III} \times \frac{U_{III}}{U_{II}} \]  

(5.5)

\section{Parallel operation of transformers in three-phase systems}

In this clause, parallel operation means direct terminal-to-terminal connection between transformers in the same installations. Only two-winding transformers are considered. The logic is also applicable to banks of three single-phase transformers.

For successful parallel operation, the transformers require:

- the same phase-angle relation – clock-hour number (additional possible combinations are mentioned below);
- the same ratio with some tolerance and similar tapping range;
- the same relative short-circuit impedance – percentage impedance – with some tolerance. This also means that the variation of relative impedance across the tapping range should be similar for the two transformers.

These three conditions are elaborated further in the following subclauses.

It is important that the tender specification for a transformer which is intended for parallel operation with a specific existing transformer contain the existing transformer information. Some warnings are prudent in this connection.
It is not advisable to combine transformers of widely different power rating (say, more than 1:2). The natural relative impedance for optimal designs varies with the size of the transformer.

Transformers built according to different design concepts are likely to present different impedance levels and different variation trends across the tapping range (see 4.4).

The consequences of a small mismatch of data should not be overestimated. It is not necessary, for example, to provide precisely the same tapping voltages on two parallel transformers. The tapping step is usually so small that staggered taps permit reasonable operation. However, care should be taken where tapping steps are large (see 2.6 and 3.6).

Specified and guaranteed ratio and impedance parameters are subject to tolerances, according to table 1 and clause 9 of IEC 67001-1. The possibility of tightening up the tolerances in particular cases with special reference to parallel operation is given in table 1, note 7 of IEC 67001-1.

In practice, a mismatch of relative loading of no more than about 0.1 % between two transformers of non-identical designs should be regarded as reasonable.

Matching three-phase connections and phase-angle relations

The common three-phase transformers connections shown in figure 12 are derived from annex D of IEC 67001-1. In each block, one or two clock numbers are indicated by permutation. This means for example that if the secondary terminals of a transformer with clock number 1 were quite simply re-named by cyclic permutation (II becomes I, III becomes II and I becomes III), then the phase displacement is changed by 21 electrical degrees to clock number 5. Consequently, transformers with connection clock numbers differing by 4 or 8 can be connected in parallel after cyclic permutation of the connections to the line on either side of the transformer.

It is even possible to combine transformers having clock numbers 1 or 5 with transformers having clock numbers 11 or 7 by reversing the phase sequence on both sides of either transformer.

Parallel connections of Dyn and Yzn are not recommended because of different zero sequence impedance properties.
### Figure 11 – Common three-phase transformer connections, and some artificial paralleling possibilities

1. **Difference in ratio, circulating current**

If two transformers with slightly different ratios are energized in parallel, this will give rise to a circulating current between the transformers. The approximate magnitude of this current is assessed in the following way.
Two transformers a and b with rated power $S_a$ and $S_b$ and with relative short-circuit impedances $z_a$ and $z_b$ are energized in parallel at no load from either side. The difference between the induced no-load voltages $U_a$ and $U_b$ on the opposite side of the transformers is expressed as a fraction $p$ of the average voltage, which is assumed to be approximately equal to the rated voltage $U_r$:

$$p = \frac{(U_a - U_b)}{U_a + U_b} \approx \frac{U_a - U_b}{U_r}$$

This voltage difference drives the circulating current through the sum of the two parallel transformer impedances. As these are mainly inductive, the circulating current is also inductive.

The circulating current $I_c$, and the corresponding reactive power $Q_c$, expressed as a fraction of rated current $I_r$ and rated power $S_r$ of the respective transformers, will be, approximately

$$I_c = \frac{Q_c}{S_{ra}} \approx \frac{p}{z_a + \frac{S_{ra}}{S_{rb}} \times z_b}$$

$$I_c = \frac{Q_c}{S_{rb}} \approx \frac{(-p)}{z_b + \frac{S_{rb}}{S_{ra}} \times z_a}$$

If both transformers have the same rated power and the same relative short-circuit impedance, these expressions simplify to:

$$\pm \frac{p}{\tau z}$$

**Example:**

When $p = \cdot \cdot \cdot$ of the rated voltage, and $z = \cdot \cdot \cdot$ per unit (p.u.), the circulating current will be $\cdot \cdot \cdot$ of the rated current. This inductive current combines vectorially with the load current. The arithmetic difference between the resulting currents in the two transformers becomes quite small as long as the load power factor is relatively high. The condition is indeed less difficult than often assumed.

The analysis indicates the order of magnitude of current which flows while tapchangers of two parallel-connected transformers are staggered one step during a leader-follower tapchanging operation.

Under certain conditions, staggered tappings may even be deliberately utilized in order to compensate the reactive part of a circulating current caused by unlike short-circuit impedance values (see the following subclause).

### \(9.7\) Unequal short-circuit impedances

When two transformers have equal short-circuit impedances, this means that they present the same voltage drop for equal per unit loading (equal load current in per cent of rated current, or equal load power in per cent of rated power). If connected in parallel, they will share the load in proportion to their relative rated power values.

When transformers with unequal short-circuit impedance values are connected in parallel, the transformer with a lower impedance value will take up a higher percentage of its rated power than will the higher impedance transformer, so that the absolute voltage drop will be
the same for both transformers. This may cause marginally increased combined power loss for the transformation but, above all, it may restrict the loadability of the installation.
The unbalance is estimated as follows.

As in the previous subclause, let a and b be two transformers with rated power $S_{ra}$ and $S_{rb}$, and relative impedances $z_a$ and $z_b$. The transformers have equal ratio. The load to be shared is $S$. The units will take up $S_a$ and $S_b$ respectively:

$$\frac{S_a}{S_{ra}} = \frac{S}{S_{ra} + \frac{z_a}{z_b} \times S_{rb}}; \quad \frac{S_b}{S_{rb}} = \frac{S}{S_{rb} + \frac{z_b}{z_a} \times S_{ra}}$$

(89)

Example:

Transformer a: $S_{ra} = 10 \text{ MVA}, \quad z_a = 10\%$

b: $S_{rb} = 20 \text{ MVA}, \quad z_b = 15\%$

The combined load is $S = 70\%$ of the sum of the rated power figures of the transformers.

The actual loading however becomes:

$$\frac{S_a}{S_{ra}} = 1 \times 10 \approx 1 \text{ MVA}; \quad \frac{S_b}{S_{rb}} = 0.8 \times 20 \approx 18 \text{ MVA}$$

$S_a = 1 \times 10 \approx 10 \text{ MVA}; \quad S_b = 0.8 \times 20 \approx 18 \text{ MVA}$

Transformer a is fully loaded, while transformer b has only taken up $8\%$ of its rated power.

The theoretical loadability of this combination would thus be reduced about $10\%$ in comparison with what would be possible if the load had been ideally shared. This should, however, be regarded as quite reasonable, if it is a question of combining already existing transformers. According to IEC 6700-1, the tolerance on specified short-circuit reactance for a new transformer at the principal tapping is $5.7\%$ to $1\%$ of the declared value. For other tappings, the tolerance is wider.

The influence on combined load loss of the mismatch between the two parallel transformers is practically negligible.

Sometimes there is a possibility of partially compensating the effect of unlike short-circuit impedance values by deliberately staggering the tapchangers. This compensation, however, works only on the reactive component of load current, and is therefore effective only when the power factor is relatively low.

\section*{Variation of short-circuit impedance across the tapping range, influence of winding arrangement}

Subclause 5.5 of IEC 6700-1 discusses different ways of specifying impedance in an enquiry for a transformer with tapchanger regulation. It is pointed out in a note that a complete specification of the impedance variation across the tapping range will dictate the arrangement of windings in a quite restrictive manner.

Tolerances on impedance values for individual tappings are dealt with in IEC 6700-1 (clause 9 and table 1). There is also an alternative method of specifying tappings by boundaries around a permitted range which may leave more freedom in the practical case (see annex C of IEC 6700-1).
Figure 22 illustrates the typical variation pattern of the series impedance for a transformer with a separate tapping winding in series with the high-voltage main winding. The tapping winding may be placed either on the same side of the low-voltage winding as the main winding, figure 22a or on the opposite side, figure 22b.

Figure 22 is equally valid for a core-form transformer with concentric windings and for a shell-form transformer with axially stacked windings. Compare, for example, two core-form transformers with the following winding arrangement, from the core outwards:

- low-voltage – high-voltage main winding – tapping winding, see figure 22a;
- tapping winding – low-voltage – high-voltage main winding, see figure 22b.

Both arrangements are in use and manufacturers probably standardize on either of them as their preferred solution in a certain range of rated power and voltage. But the two alternatives have opposite impedance variation trends across the tapping range.

Variation of active turns in tapping winding causes the variation of the leakage flux pattern and of the corresponding leakage permeance of the transformer. This latter, when the tapping winding is adjacent to the main HV winding (figure 22a), increases when the regulating turns are increased in sum with the main winding and decreases when the regulating turns are increased in subtraction. When the tapping winding is not adjacent to the main HV winding (figure 22b), the effect of the turn variation on the trend of leakage permeance is exactly reversed. Moreover, the leakage permeance variation is small for the arrangement of figure 22a and more important for the arrangement of figure 22b.

The percentage short-circuit impedance (and its variation) reflects the leakage permeance only (and its variation) and must be independent of the side considered, assuming a constant rated power as reference. Also the absolute short-circuit impedance variation (in ohms), seen from the constant turn winding (LV), reflect the leakage permeance variation only.

The above statements are represented in curves 1 of figures 22a and 22b. There is a small variation for the figure 22a arrangement and a more pronounced variation for the arrangement in figure 22b.

The absolute short-circuit variation (in ohms), seen from the variable turn winding (HV), reflects both the leakage permeance variation and the square of the active turns, since it is derived from a reference value, proportional to the square of the active turns. With the arrangement in figure 22a, the variation of the square of the active turns has the same trend of the leakage permeance variation and therefore the total variation is enhanced. With the arrangement in figure 22b, on the contrary, the variation of the active turns has the inverse trend of permeance variation and therefore the total variation is attenuated. This is represented by curves 2 of figures 22a and 22b.

These matters are of importance particularly during the detailed analysis of through-fault short-circuit current, depending on actual transformer tappings.
Figure ٢٢a – LV winding external to HV and regulating winding

Figure ٢٢b – LV winding between HV and regulating winding

Figure ٢٢ – Variation of impedance across the tapping range depending on location of regulating winding

- Impedance variation in ohms from the untapped winding side (low-voltage)
- Impedance variation in ohms from the tapped winding side (high-voltage)

Calculation of voltage drop for a specified load, three-winding transformer load loss

Introduction: the need for voltage drop calculation

The IEC definitions concerning rated power and rated voltage of a transformer imply that rated power is input power, and that the service voltage applied to the input terminals for the active power (the primary terminals) should not, in principle, exceed the rated voltage (see the note in ٢٢١ of IEC ٦٧٠٠٦١).
The maximum output voltage under load is therefore a rated voltage (or tapping voltage) minus a voltage drop. The output power at rated current and rated input voltage is, in principle, the rated power minus the power consumption in the transformer (active power loss and reactive power).

In North America, the MVA rating is based on maintaining the rated secondary voltage by impressing on the primary winding the voltage necessary to compensate for the voltage drop across the transformer at rated secondary current and at a lagging power factor of 0.8 % or higher. Using this method, the practical difference in calculating the voltage drop is minor. Equations (17) show this contradiction.

The determination of the corresponding rated voltage or tapping voltage which is necessary to meet a specific output voltage at a specific loading, therefore involves a calculation of voltage drop, using known or estimated figures of transformer short-circuit impedance. This subclause derives the expressions which conform to the IEC definitions of ratings and losses of the transformer.

The short-circuit impedance and equivalent diagram of a two-winding transformer

The voltage drop of a transformer is defined as the arithmetic difference between the no-load voltage of a winding and the voltage developed at its terminals at a specified load; see 2.7.3 of IEC 67006-1, and IEC 6368 (124), definition 70-30. Unless otherwise stated, the voltage across the other (primary) winding is taken as rated voltage (or tapping voltage, as the case may be).

The conventional equivalent circuit of the transformer contains a linear series impedance (for a multi-winding transformer, an impedance network) across which the voltage drop develops. The series impedance is identified with the short-circuit impedance, measured in a routine test of the transformer. See the measurement of short-circuit impedance and load loss in 4.01 of IEC 67006-1. The voltage drop is independent of the actual voltage as the voltage-dependent magnetizing current is neglected in voltage drop calculations.

The test permits separation of the series impedance into a resistance, representing the load loss, and a reactance:

\[ Z = R + jX \]

Conventionally, the impedance is expressed in relative form, as a fraction of the reference impedance \( Z_{\text{ref}} \) of the transformer and expressed in per cent. The relative impedance is written:

\[ z = r + jx \quad \text{where} \quad z = \frac{Z}{Z_{\text{ref}}} \]

and \( Z_{\text{ref}} \), in turn is \( Z_{\text{ref}} = \frac{U_{\text{ref}}}{S_{\text{ref}}} \)

\( U_{\text{ref}} \) is the voltage of the winding, to which \( Z \) and \( Z_{\text{ref}} \) are referred. (Unless otherwise specified, it is the rated voltage of the winding but, if a particular tapping other than the principal tapping is referred to, the reference voltage may be the tapping voltage instead.) \( S_{\text{ref}} \) is the reference value of power for the pair of windings involved. This is normally the rated power of either winding of the pair but the reference value should always be noted to avoid misunderstanding.
For a three-phase transformer, \( Z \) and \( Z_{\text{ref}} \) are impedances per phase (equivalent star connection); see 7.2.1 of IEC 67006-1. According to IEC 67006-1, the relative, or percentage, value of impedance is presupposed to be one and the same, regardless which of the two involved windings is energized and which is short-circuited in the test.

\[ \text{Description of the load} \]

The load on the transformer is expressed as an arbitrary value \( S \) of apparent power (not identified with rated power), and a phase angle \( \phi \), or as separate values of active and reactive load, \( P \) and \( Q \). Together with this is given the terminal voltage, \( U_\tau \), at which the load is delivered on the secondary side of the transformer.

The notation follows conventions for complex numbers in polar form (absolute value |\( S \)| and argument \( \phi \)) or with real and imaginary parts \( P \) and \( jQ \), as shown below:

\[
S = |S|, \phi = P + jQ = |S|(\cos \phi + j\sin \phi)
\]  

This load may be expressed as a load impedance \( Z_L \) (ohms per phase):

\[
Z_L = \frac{|U|}{S}(\cos \phi + j\sin \phi)
\]

The load may also be described as a load current \( I_\tau \), together with the phase angle \( \phi \) of the load (the phase angle between terminal voltage \( U_\tau \) and current \( I_\tau \)).

\[
|U_\tau| = |I_\tau| + \phi
\]

In the single-line diagram representing symmetrical three-phase loading, \( U_\tau \) and \( U_\tau \) are to be replaced by \( U_\tau / \sqrt{3} \) et \( U_\tau / \sqrt{3} \) (for equivalent star connection). However, this makes no change in the following text.

\[ \text{The voltage drop equations} \]

The transformer at no load has a secondary voltage \( U_\tau \). With the load connected, the secondary terminal voltage changes to \( U_\tau \).
Using the impedance notation, the relation between $U_r$ and $U_{r_1}$ becomes (see figure ٩٠)

$$\frac{U_r}{U_{r_1}} = \frac{Z_L}{Z_L + Z_T}$$

(٩٠)

The voltage drop is defined as the arithmetic difference

$$\Delta U_r = |U_{r_1} | - |U_r |$$

(٩١)

Combine with equation (٩١):

$$\Delta U_r = |U_{r_1} | \cdot \left|\frac{Z_L + Z_T}{Z_L} \right| ; \quad \Delta U_r = |U_{r_1} | \cdot \left|\frac{-Z_L}{Z_L + Z_T} \right|$$

(٩٢)

Insert the components of the transformer impedance in the expression of the load impedance according to equation (٩٣):

$$\left( \frac{Z_L + Z_T}{Z_L} \right) = \left( 1 + \frac{Z_T}{Z_L} \right) = \left( 1 + \frac{S}{U_{r_1}^*} \right) \left( R_T + j X_T \right) \left( \cos \phi - j \sin \phi \right)$$

$$= \left( 1 + \frac{S}{U_{r_1}^*} X_T \sin \phi + R_T \cos \phi \right) + j \left( \frac{S}{U_{r_1}^*} X_T \cos \phi - R_T \sin \phi \right) = (1 + A + jB)$$

(٩٣)

The moduli of the expression, and of its inverse, are

$$\left( (1 + A)^* + B^* \right)^* = 1 + A + \frac{B^*}{1} + .....$$

(٩٤)

$$\left( (1 + A)^* + B^* \right)^{-*} = 1 - A + \frac{B^*}{1} + .....$$

(٩٥)

The voltage drop is therefore

$$\Delta U_r = |U_{r_1} | \left( A + \frac{B^*}{1} + ..... \right) ; \quad \Delta U_r = |U_{r_1} | \left( A - A^* + \frac{B^*}{1} + ..... \right)$$

(٩٦)

The voltage drop is a first order difference and the expressions in the parentheses in turn differ by a second order difference, depending on whether the calculation is based on the terminal voltage $U_r$ or the equivalent no-load voltage $U_{r_1}$. This refinement is usually negligible (see the numerical example in the following subclause).

The first term, $A$, in the expansion is geometrically interpreted as the projection of the voltage drop phasor $I_r Z_T$ on the $U_r$ phasor (see figure ٩٢).
The arithmetic voltage drop $\Delta U_2$, as defined, is usually considerably less than the absolute value $|I_2 Z_T|$ of the vectorial voltage drop $I_2 Z_T$ which also represents the phase angle difference between $U_2$ and $U_1$. The absolute value of the vectorial voltage drop is independent of load phase angle $\phi$.

**Figure 42** – Phasor diagram showing vectorial and arithmetic voltage drop

For all large power transformers, the reactive part of the series impedance is much larger than the resistive part. $X_T$ is typically 5% to 10%; $R_T$ is less than 1%.

If the phase angle is negative (capacitive or partly capacitive loading), the voltages drop may come out negative. The secondary voltage then rises above its no-load value when the load is connected.

\[ Z_T = R_T + j X_T = \frac{Z_T}{Z_r} \times 100\% = \frac{S_r}{U_r} \times Z_T \times 100\% \]

Assume that the reference voltage $U_r$ (rated voltage or tapping voltage) is at least approximately equal to the actual secondary voltage $U_2$.

Then the voltage ratio expression in equation (5x) may be rewritten

\[
\left( \frac{Z_L + Z_T}{Z_L} \right) = \left( 1 + \frac{S}{S_r} \left( \frac{x_T}{\sin \phi} + \frac{r_T}{\cos \phi} \right) \right) \left( 1 + \frac{S}{S_r} \left( \frac{x_T}{\cos \phi} + \frac{r_T}{\sin \phi} \right) \right) \]

The ratio $S/S_r$ is the ratio of actual loading over rated power and this is also the ratio of actual load current over rated current.
The following representative numerical example illustrates the relative magnitudes of the different terms.

**Example:**

\[ \frac{S}{S_r} = 0.5, \quad x = 15\%, \quad r = 7\%, \quad \cos \phi = \text{inductive} \]

If \( \cos \phi = 0.8, \sin \phi = 0.6 \). The voltage ratio expression becomes

\[ 1 + 0.05 \times (0.15 + 0.8 \times 0.6 + 0.8 \times 0.6 + 0.6 \times 0.8) + j \times 0.05 \times (0.15 \times 0.8 \times 0.6 - 0.6 \times 0.8) \]

\[ = 1 + 0.048 + j \times 0.02 \]

The percentage voltage drop becomes

\[ \frac{U_U}{U_r} = 1 + 0.05 \times \left(0.048 + j \times 0.02\right) = 1.05 \times \text{or} \ 0\% \]

Observe that the first approximation of the voltage drop, taking the term A directly, would give the result 4.8%, as against 0.5% for the complete calculation, an approximation which is satisfactory for practical purposes and is better than the overall accuracy of measurements in service.

---

**V.7** *Equivalent diagram for multi-winding transformers, T-equivalent impedance elements for a three-winding transformer*

The equivalent single-line diagram for a two-winding transformer, which is in agreement with the definitions and tests of IEC 6700-1, is presented in figure 32. An analogous extension to a multi-winding transformer is shown in figure 52. This figure indicates a pair of primary terminals, at which active power is supplied to the transformer, and a set of output windings: secondary, tertiary... The diagram contains the magnetizing admittance element placed at the input terminals. Ideal transformers representing the no-load turns ratios between different windings are connected to the linear network of series impedances which for an n-winding transformer contains \( \frac{n(n-1)}{2} \) independent elements.

![Figure 52 – Single-line equivalent diagram of multi-winding transformer](image)
This equivalent diagram corresponds to a set of linear voltage drop equations. The individual impedance elements can in principle be evaluated from, for example, the set of independent measurements of short-circuit impedances for all two-winding combinations.

In a three-winding transformer, the series impedance network has three elements. It is known from circuit theory how a triangle configuration of three such elements can always be transformed into a star, or T, configuration and vice versa. The T configuration is suitable for system analysis e.g. of the flow of reactive power through a three-winding transformer. The three T-elements are evaluated from the parameters of two-winding combinations as shown in figure 62.

\[
\begin{align*}
Z_A &= Z_{AB} - Z_{BC} + Z_{CA} \\
Z_B &= Z_{BC} - Z_{CA} + Z_{AB} \\
Z_C &= Z_{CA} - Z_{AB} + Z_{BC}
\end{align*}
\]

**Figure 62 – Three-winding transformer impedances**

The star elements are algebraic combinations of physical quantities. It is possible, and not absurd, that one of the elements may come out with negative reactance. This is typically the case for an element associated with a winding that is physically placed between the other two.

The separation of resistances into star elements implies that the measured load losses for the different two-winding combinations are allocated to the individual windings. This procedure is conventionally accepted, but has doubtful accuracy for large transformers where eddy losses in windings and stray flux losses in other parts are of considerable importance. These loss components do not lend themselves well to linear combinations according to the simple procedure indicated here.

**Allocation of load losses to individual windings in three-winding transformers**

**The general case**

The preceding subclause describes how series impedances in a three-winding transformer are measured as two-winding combinations and then allocated to each of the three separate windings by a linear transformation to form a star configuration. It must be repeated that this is only a mathematical trick; there is no such thing, physically, as the series impedance of an individual winding since the impedances only exist between windings.
A corresponding procedure is applied to the load loss of three-winding transformers for a specific loading combination. Let us in this context refer to the windings as H (high-voltage), X (intermediate voltage), T (low-voltage tertiary). Figures of load loss are determined together with series impedance for the three possible two-winding combinations:

\[ P_{HX}, P_{HT}, P_{XT} \]

These measurements may have been taken with currents in the windings corresponding to different values of reference power, because the tertiary winding usually has a lower assigned value of rated power. For the following transformation it is convenient to bring the measurements to a common basis of reference power, \( S' \), proportioning by the square of the current:

\[ P_{HX}', P_{HT}', P_{XT}' \]

The reference currents in the respective windings are

\[ I'_H, I'_X, I'_T \]

The allocation of losses to the individual windings is made by the transformation

\[
\begin{align*}
P'_H &= \frac{1}{I_H} (P_{HX}' + P_{HT}' - P_{XT}') \\
P'_X &= \frac{1}{I_X} (P_{HX}' - P_{HT}' + P_{XT}') \\
P'_T &= \frac{1}{I_T} (-P_{HX}' + P_{HT}' + P_{XT}')
\end{align*}
\]

For a given load combination, with actual currents in the windings equal to \( I_H, I_X, I_T \), the resulting loss for each winding is again proportional to the square of current (or power) from the reference figures above. The resulting load loss \( P_K \) for the whole transformer is the sum of these three single-winding allocated loss figures:

\[
P_K = (I_H / I'_H)^{\gamma} \times P'_H + (I_X / I'_X)^{\gamma} \times P'_X + (I_T / I'_T)^{\gamma} \times P'_T
\]

In terms of apparent power in the windings the formula reads

\[
P_K = (S_H / S')^{\gamma} \times P'_H + (S_X / S')^{\gamma} \times P'_X + (S_T / S')^{\gamma} \times P'_T
\]

Note that power figures in the formula above are interpreted in terms of actual current and corresponding no-load voltage for windings with outgoing load, not actual voltage on the terminals, incorporating voltage drop in the transformer.
The algorithm for calculation of the combined three-winding loss is valid for single-phase and three-phase transformers, and not only for separate-winding transformers but also a three-winding transformer where two windings are auto-connected.

\[ \gamma, \eta, \tau \]

The case of an auto-connected transformer

The allocation of the load losses to individual physical windings, in order to estimate temperature rise of individual windings under a specific load combination, is more complicated for an auto-connected transformer. This is because the physical windings of the auto-connected pair are not identical with the windings in the sense of the standards. We must study the series winding \( S \) and the common winding \( C \), the physically separate parts of the auto-connected pair instead of the high-voltage and the intermediate voltage windings.

The definition of the reduction factor of the auto-connection is recalled (see \( \tau, \gamma \) and \( \tau, \gamma \)):

\[ \alpha = \frac{U_H - U_X}{U_H} = \frac{l_X - l_H}{l_X} \]

This is the ratio between the true apparent power of the physical windings \( S \) or \( C \), and the throughput power between the formal windings \( H \) and \( X \).

Consider the two-winding case when reference power \( S' \) is transformed between windings \( H \) and \( X \).

The series winding has voltage \( U_H - U_X = \alpha U_H \), while its current is \( I'_H \).

The voltage across the common winding is \( U_X \) and its reference current is

\[ l'_C = l'_X - l'_H = \alpha l'_X \]

The equivalent reference power for the auto-connected physical windings is not \( S' \), but \( \alpha S' \).

The three two-winding loss combinations, all referred to throughput power \( S' \) are illustrated in figure \( \tau, \gamma \). The actual current in winding \( C \) is different in the three cases.
It is obvious from the above that the two-winding losses may be allocated to the respective windings by the relations

\[
\begin{align*}
P'_{HX} &= P'_S + P'_C \\
PPH_1 &= P'_S + \left(\frac{1-\alpha}{\alpha}\right)P'_C + P'_T \\
PX_H &= \left(-\frac{1}{\alpha}\right)P'_C + P'_T
\end{align*}
\]

\[(\gamma^2)\] (\gamma^3)

The inversion of this set of equations is

\[
\begin{align*}
PPS & = \frac{1}{\gamma}\left((\gamma-\alpha)P'_{HX} + \alpha P'_{HT} - \alpha P'_{XT}\right) \\
PPH_2 & = \frac{1}{\gamma}\left(\alpha P'_{HX} - \alpha P'_{HT} + \alpha P'_{XT}\right) \\
PST & = \frac{1}{\gamma}\left(\frac{1}{\alpha}P'_{HX} + \frac{1}{\alpha}P'_{HT} + \frac{\gamma - \alpha}{\alpha} P'_{XT}\right)
\end{align*}
\]

\[(\gamma^4)\]
The load loss for each winding during a specific loading case, characterized by currents $I_H$, $I_X$, $I_T$ is then written

$$P_S = P'_S \left( \frac{I_H}{I'_H} \right) \gamma; \quad P_C = P'_C \left( \frac{I_C}{I'_C} \right) \gamma; \quad P_T = P'_T \left( \frac{I_T}{I'_T} \right) \gamma; \quad (\gamma^2)$$

In order to assess the loss in the common winding, it is necessary to calculate the current in that winding:

$$|I_C| = |I_X - I_H|$$

$I_X$ and $I_H$ are not normally in phase during a three-phase loading case with an independent load on the tertiary winding.

But the loading case is conventionally described in terms of values of apparent power on the terminals of windings $H$, $X$ and $T$. The following expressions are used:

$$P_S = P'_S \left( \frac{S_H}{S'_H} \right) \gamma; \quad P_C = P'_C \left( \frac{S_C}{\alpha S'_C} \right) \gamma; \quad P_T = P'_T \left( \frac{S_T}{S'_T} \right) \gamma; \quad (\gamma^7)$$

However, here we need an expression for $S_C$ in the general case. In the following equations, current and power values are vectorial, while turns numbers or equivalent no-load voltages are scaler constants.

The power sum is zero:

$$\mathbf{S}_H + \mathbf{S}_X + \mathbf{S}_T = 0 \quad (\gamma^7)$$

Using equivalent no-load voltages:

$$\bar{I}_H U_H + \bar{I}_X U_X + \bar{I}_T U_T = 0 \quad (\gamma^8)$$

There is also ampere-turn balance.

$$\bar{I}_H n_S + \bar{I}_C n_C + \bar{I}_T n_T = 0 \quad (\gamma^9)$$

But the numbers of turns are proportional to the respective rated, or tapping voltages. Therefore:

$$\bar{I}_H (U_H - U_X) + \bar{I}_C U_X + \bar{I}_T U_T = 0; \quad U_H - U_X = \alpha U_H \quad (\gamma^{14})$$
Using the cosine theorem for the triangles:

\[
\begin{align*}
(I_X U_X) &= (I_H U_H) + (I_T U_T) - \gamma I_H U_H I_T U_T \cos \phi \\
(I_C U_C) &= \alpha (I_H U_H) + (I_T U_T) - \gamma \alpha I_H U_H I_T U_T \cos \phi
\end{align*}
\]

Eliminating the angle function between the equations:

\[
S_C = (1 - \alpha) \left( S_T - \alpha S_H \right) + \alpha S_X
\]

This relation contains only the absolute values of powers. Phase relationships are implicit in the deduced expression.

The allocated loss in winding C is then, finally

\[
P_C = P'_C \left( 1 - \alpha \right) \frac{S_T - \alpha S_H + \alpha S_X}{\alpha (S')^T} \quad (\text{\textsuperscript{\textsection}3)}
\]

If \(|S_H| = |S_X| = |S_T|\), the expression simplifies to

\[
P_C = P'_C \left( \frac{S_H - \alpha S_T}{S'} \right) \quad (\text{\textsuperscript{\textsection}4)}
\]

\textsuperscript{\textsection}1

Example of calculation of voltage drop and load loss for a three-winding transformer

This example is intended to demonstrate, step by step, how such calculations may be performed in a simple way by suitable approximations.

Investigate a loading case for a three-winding, separate-winding transformer, having the following power ratings for the three windings:

- primary winding (I) \(8\) MVA;
- secondary winding (II) \(8\) MVA;
- tertiary winding (III) \(51\) MVA.

This transformer is loaded as follows.
The input voltage to the primary winding is assumed to be equal to the rated voltage of that winding. The secondary winding delivers 57 MVA at an inductive power factor of 8.0.

The tertiary winding is loaded with a fixed capacitor bank, which is rated 51 MVA at a voltage equal to the rated voltage of the tertiary winding.

It is necessary to calculate the voltage drops through the transformer in order to find the output voltages on terminals (II) and (III). However as the problem is formulated, the load currents are not given, hence they have to be calculated.

The load on the secondary winding is specified without reference to the actual service voltage. Therefore the load current, and consequently the voltage drop, must be assumed to be inversely proportional to the terminal voltage.

The tertiary load, on the other hand, is assumed to be a capacitor bank of fixed impedance. For this load, the current increases in proportion to the terminal voltage and the reactive power generated by the bank rises with the square of the voltage.

The transformer test report contains the following values of two-winding short-circuit percentage impedances, all referred to 0.8 MVA, and corresponding load loss referred to different ratings. Table 2 also shows these loss figures expressed as percentages on the common basis of 0.8 MVA.

<table>
<thead>
<tr>
<th>Combination</th>
<th>Impedance ( % )</th>
<th>Load loss/ reference ( \text{kW /} ) 0.8 MVA</th>
<th>Load loss/ 0.8 MVA ( % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(I) and (II)</td>
<td>11.0</td>
<td>3.0 kW / 0.8 MVA</td>
<td>3.77 0</td>
</tr>
<tr>
<td>(I) and (III)</td>
<td>13.2</td>
<td>2.0 kW / 0.8 MVA</td>
<td>2.111</td>
</tr>
<tr>
<td>(II) and (III)</td>
<td>27.3</td>
<td>2.0 kW / 0.8 MVA</td>
<td>2.889</td>
</tr>
</tbody>
</table>

The two-winding parameters are converted to an equivalent star configuration. The values for total short-circuit impedance and for loss resistance, separately, are noted in figure 14.

Note however that this is a purely mathematical representation of the transformer as a black box. It reflects the behaviour of the transformer as measurable on its terminals, but the model is not a physical description of the unit in terms of its different windings, etc. The junction T is fictitious. One of the reactance elements comes out negative (which actually indicates that the winding connected to terminal (I) is placed between the other two in the transformer).
In the following, a method with successive approximations will be used. This lends itself to
calculation by hand and illustrates the relative importance of different parameters.

**First approximation**: Assume rated voltage on the output terminals and calculate voltage
drop and resulting input power to the transformer.

**Second approximation**: Correct the output voltages with the first approximation voltage
drop figures and modify the load parameters accordingly (loading currents are implicit,
depending on the actual voltage). Obtain corrected load power and voltage drop values.

For these approximate calculations, the short-circuit impedance will be regarded as mainly
reactive i.e. \( Z \approx X \). The losses are disregarded in comparison to the active load power.

For each branch of the equivalent-star impedance network, the following relations are
used. Per unit values are indicated by brackets:

\[
[i] = \left( \frac{S}{S_r} \right) \left( \frac{U_r}{U} \right)
\]

Reactive power consumption

\([x][i]^*\)

Power loss

\([\rho][i]^*\)

Arithmetic voltage drop

\(\sin \phi [x][i]\)

\[= \left( \frac{Q}{S} \right) [x] \left( \frac{S}{S_r} \right) \left( \frac{U_r}{U} \right) = [x] \left( \frac{Q}{S_r} \right) \left( \frac{U_r}{U} \right)\]

Note that in these equations the reactive load power \( Q \) is positive for reactive loading
(branch II), and negative for capacitive loading (branch III). Per unit short-circuit reactance
\([x]\) is negative for branch I and positive for branches II and III, see figure 92.26.

**Figure 92 – Star-equivalent short-circuit impedances and
star-equivalent load loss components**
First approximation

Branch (II)
- Apparent power: $51,390 = 5.8$ MVA per unit
- Active load: $8 \times 51 = 0.6$ MW
- Reactive load: $6 \times 51 = 54$ MVA r
- Per unit current: $1.1250 \times (1.1250)^2 = 0.11$ per unit
- Voltage drop: $0.6 \times 0.1250 \times 1.1250 = 0.06$ per unit
- Reactive consumption: $0.1250 \times (1.1250)^2 = 0.11$ per unit

Branch (III)
- Apparent power: $51,390$ per unit (capacitive) $= -51,390$ per unit
- Per unit current: $-1.1250$
- Voltage drop: $-0.1250 \times -1.1250 = -0.14$ per unit
- Reactive consumption: $-0.1250 \times (-1.1250)^2 = -0.14$ per unit

Node T, combined power flow
- To branch (II): $0.6$ MW and $(4.9 + 4.9)$ MVA r
- To branch (III): $(-1.1 + 4.9)$ MVA r
- Sum, from (I): $0.6$ MW and $9.9$ MVA r is equivalent to $7.1$ MVA
- Per unit current in branch (I): $4.99$

Branch (I)
- $\sin \phi$: $39.3/\sqrt{1.1} = 0.944$
- Voltage drop: $0.944 \times (1.301) \times 4.99 = (1.301) \times 4.99$ per unit
- Reactive consumption: $-1.301 \times (1.301)^2 \times 0.8 = -1$ MVA r

Resulting voltages
- Node T: $1.301 + 1.301 = 1.301$ per unit
- Terminal (II): $1.301 - 0.171 = 0.930$ per unit
- Terminal (III): $1.301 + 0.28 = 1.58$ per unit

Second approximation

The terminal voltages according to the first approximation are introduced instead of rated voltages ($1 \cdots 1$ per unit):

Branch (II)
- $[j] = 1.1250/1.1250 = 1 \cdots 1$

The reactive power consumption and arithmetic voltage drop change to
- reactive consumption: $0.1250 \times (1 \cdots 1)^2 \times 0.8 = 1 \cdots 1$ MVA r;
- voltage drop, branch (II): $0.6 \times 0.1250 \times 1 \cdots 1 = 0.06$ per unit.
The tertiary voltage rises, when the capacitor bank is connected (negative voltage drop). The bank has a fixed impedance. The load current rises in proportion to the voltage, and the generated reactive power rises by the square of the voltage. The new values become the following:

**Branch (III)**

- Per unit current: $5781,0 \times 630,1 = 491,0$
- Capacitor loading: $(1032)^7 \times 10 = 1741 \text{ MVA}_r$
- Reactive consumption: $630,1 \times (-0,94)^7 \times 5 = 42 \text{ MVA}_r$
- Voltage drop: $6147 \times -0,94 = -5510,0 \text{ per unit}$

**Node T, combined power flow**

- Sum of reactive power: $54 + 101 - 17 = 39 \text{ MVA}_r$
- Sum from (I): $60 \text{ MW and } 39,4 \text{ MVA}_r = 71,8 \text{ MVA}$
- Per unit current in branch (I): $0,997$

**Branch (I)**

- $\sin \phi = 39,4/71,8 \approx 0,549$
- Voltage drop: $742 \times (0,105) \times 798 = -100,0 \text{ per unit}$
- Reactive consumption: $742 \times (0,997)^7 \times 8 = -1 \text{ MVA}_r$

The second estimate of voltages becomes

- terminal (II): $1001 - 570 = 431 \text{ per unit}$
- terminal (III): $800,1 + 920,0 = 730,1 \text{ per unit}$

If another iteration would be made, the change would be only of the order of $100,0$. At that level of numerical accuracy, however, the approximations in the basic model of the transformer and in the calculation procedure would no longer be negligible.

For comparison, a computer calculation with standard software (complex number algebra) has given

$$U_{(II)} = 0,93 \text{ per unit}; U_{(III)} = 1,01 \text{ per unit}.$$ 

Combined load loss, using the hand-calculated branch current estimates:

- branch (I): $990,0 \times (0,997)^7 = 970,0 \%$
- branch (II): $772,0 \times (1,01)^7 = 772 \%$
- branch (III): $316,0 \times (0,105)^7 = 320 \%$

The sum is $973,0 \% = 3,3 \text{ kW, say 3,1 kW}$.

---

**Example of calculation of combined load loss and allocation of losses to individual windings in an auto-connected three-winding transformer**

A transformer is specified as $530/530/121 \text{ MVA YNauto d ONAF}$. Rated voltages are $380$, with regulation $137/137 \text{ kV}$. 

Under ONAN cooling (fans not running), a heat run test is specified with 5% of rated power on all terminals: 11.09/11.09/1 MVA, and with the tapped 13.1 kV winding connected on the minimum tapping, 0.33 kV. What value of combined load loss is to be applied for the heat run test and what is the allocation of losses to the individual windings of the transformer?

Two-winding measurements have given the following results, recalculated to reference temperature:

- \( H,X \): 13.1/13.1 kV, 50 MVA, 1118 kW
- \( H,T \): 13.1/13.1 kV, 120 MVA, 1235 kW (50 MVA \( \rightarrow \) 1118 kW)
- \( X,T \): 13.1/13.1 kV, 120 MVA, 1235 kW (50 MVA \( \rightarrow \) 1118 kW)

In the last column above, the two test results involving the tertiary winding have also been recalculated to the (unrealistic) common reference power value \( S' = 50 \) MVA.

The auto-connection factor \( \alpha \) is \( (13.1 - 13.1)/13.1 = 0.0 \).

According to equations \( (\gamma \delta) \), the allocated reference loss figures for the three physical windings become

\[
\begin{align*}
PS' &= \frac{1}{\alpha} \left( 1.6 \times 11.18^2 + 1.7 \times 11.18 \times 19.89 - 1.7 \times 19.89^2 \right) = 1.784 \text{ kW} \\
PC' &= \frac{1}{\alpha} \left( 1.7 \times 11.18^2 - 1.7 \times 11.18 \times 19.89 + 1.7 \times 19.89^2 \right) = 1.784 \text{ kW} \\
PT' &= \frac{1}{\alpha} \left( -11.18 + 11.18 + 1.7 \times 19.89^2 \right) = 1.784 \text{ kW}
\end{align*}
\]

The power value to be applied for winding \( C \), the common winding, is expressed as

\[
S_C' = (1 - \alpha) \left( S_T' - \alpha S_H' \right) + \alpha S_X'
\]

The ratio \( \left( \frac{S_C'}{\alpha S'} \right)' \) becomes, numerically

\[
\begin{align*}
\left( \frac{S_C'}{\alpha S'} \right)' &= \left( \frac{1.6 \times 11.18 + 1.7 \times 19.89}{1.7 \times 19.89} \right) = 1.784
\end{align*}
\]

The allocated losses in the three physical windings, and the combined three-winding load loss become

\[
P = PS' + PC' + PT' = \left( \frac{1.784}{1.784} \right) \times 1.784 \text{ kW} + 1.784 \times 1.784 \text{ kW} + \left( \frac{1.784}{1.784} \right)^2 \times 1.784 \text{ kW} \\
= 1.784 \text{ kW} + 3.568 \text{ kW} + 3.568 \text{ kW} = 1.423 \text{ kW}
\]

**Comparison**

A straightforward calculation of combined three-winding load loss according to equations \( (\gamma \delta) \) gives
The combined loss

\[ \times \frac{5}{8292} \times 6223.5 \times \frac{5}{8292} \times 465.6 \times \frac{4}{8292} \times \frac{4}{8292} = 1.423 \, \text{kW}, \]

i.e. the same total, but without realistic subdivision between the physical windings (series, common, tertiary).

---

### Specification of rated quantities and tapping quantities

#### 8.1 Introduction

This subclause explains the relation between the service conditions for a transformer and the specified and guaranteed parameters which are called rated quantities (or tapping quantities): rated power, rated voltage, rated current. For definitions, see IEC 67006-1.

In principle, rated quantities (referring to the principal tapping of a transformer) and tapping quantities (analogous parameters for other tappings) are references for guarantees and tests regarding apparent power, voltage and current. They are not to be confused with throughput power in service and the corresponding values of voltage and current.

On the contrary, it is a relatively complicated logical procedure to determine in a correct manner, numerical values for the rated and tapping quantities which are sufficient but not unnecessarily high, from the requirements of a set of loading cases in actual service. Note that there is a whole field of combinations of service variables, while the specified quantities represent just one set of selected reference data. This set has to, however, encompass the range of allowable service conditions.

A recommended systematic procedure for this analysis is described below and illustrated by a practical example.

#### 8.2 Standardized specifications of ratings and tappings, effect of the width of the tapping range

It is not the intention here to recommend a complete analysis in every specific case when a transformer with tappings is going to be specified and purchased. This would be against the principles of sound standardization. In national standards, it is customary to provide tables of recommended ratings. These will list a series of preferred values of rated power, usually with a step of \( \frac{1}{1.529} \) or larger (see 4.3 of IEC 67006-1), combined with rated voltages and tapping ranges adapted to customary service voltage levels for the different system voltage levels in use.

The purpose of such data standardization is, among other things, to facilitate relocation of transformers to other sites, particularly when parallel operation with other units is required. The problems associated with parallel operation are dealt with separately in clause 6.
Well-organized transmission and distribution systems are designed and operated in such a way that actual variations of service voltage between light and heavy loading are quite small. This is achieved by a suitable structure of higher and lower system voltage levels and by correct management of reactive power flow. The range of tappings necessary for ratio control under normal service is therefore generally relatively limited.

It is, however, prudent to also consider abnormal conditions, when certain system components (lines, transformers) are not available. Under such emergency, or back-up service, there may be a need for a wider range of variation of transformer voltage ratio.

It may also be that different parts of a system, or adjacent systems, with one and the same standard system voltage, are, by tradition, operated at slightly different service voltage levels. In order to make the interchange of transformers possible, this may call for a wider tapping range.

However, a wide range of tappings in a transformer represents not only increased manufacturing cost and sometimes higher loss figures, but also rapidly escalating design difficulties and risk of service failure, e.g. because of transient voltage oscillations across the regulating winding. This results in requirements of increased insulation, and sometimes a more expensive tapchanger model.

Against this background of conflicting considerations, it is evident that no universal recommendation for the width of a tapping range can be given in this standard. The example which illustrates the method of analysis is just a representative practical case from an industrialized country environment and not a general recommendation.

\[\text{Procedure for the determination of rated and tapping quantities}\]

\[\text{General assumptions}\]

The transformer has two main windings. One of these windings has tappings.

Note that this terminology refers to a winding as the assembly of all turns associated with one of the voltages of the transformer (see definition in IEC 6700-1). A winding may thus consist of several separate cylinders or discs. If the winding is tapped, this means that it is reconnectible so that its effective number of turns may be altered.

A tapping in the context of IEC 6700-1 is an abstract notion, meaning a state of connection of the winding. It is identified by a specific effective number of turns and a specific tapping voltage. The tapping voltage has the same meaning for a specific tapping as the rated voltage has for the principal tapping.

How the tapped winding is designed physically is immaterial for the present discussion. It may be that the tapped part of the winding is arranged as a separate physical body, referred to as a regulating winding, and connected with the main part of the winding. The connection may be either permanent or changeable by means of the tapchanger. Terms like "linear", "plus-minus", "coarse-fine" refer to different alternatives of regulation arrangement.

It does not matter for this discussion if, in addition to the two main windings, the transformer is equipped with a stabilizing winding or a small auxiliary winding with insignificant power rating. However, a true multi-winding transformer with simultaneous power flow across two or more pairs of windings falls outside the scope of the analysis.
The direction in which active power is flowing between the two windings have to be known. If the flow is reversible, the two cases have to be taken separately.

It should also be known, before the analysis can be completed, which of the two windings is the tapped winding. In general, this is determined by technical considerations and the manufacturer's design standard rather than by the conditions of the intended installation (see also ٤.٤).

The analysis will first consider all different loading cases representing the borders of the recognized range of variation. Each case is defined by values of active and reactive load power or apparent power and power factor. There is also a range of service voltage values, at the terminals of both windings, within which it will be possible to accomplish this loading.

The secondary voltage values under all loading conditions have to be replaced by the equivalent no-load voltages by adding the voltage drop in the transformer. For this procedure, which is described in detail elsewhere in this standard, it is necessary to know, or assume, a value of the short-circuit impedance of the transformer.

In the simplest case, the specification is defined by one low-load or no-load case, and one high-load case.

In more complicated cases such as large, tailor-made system transformers exposed to different types of loading, it is recommended that a discussion is taken up with the manufacturer about the specification of the transformer, based on the recognized cases of loading, before the ratings are fixed for the tender specification. This is because a prematurely fixed specification of the transformer may lock the winding arrangement in a way that could lead to unnecessary technical difficulties or an uneconomical design.

General outline of the procedure

- For each loading case, the load current will be calculated from the specified output power and secondary service voltage. The maximum value during any of the loading cases is noted.
- The voltage drop in the transformer at the specified loading cases has first to be estimated, so that the service voltage on the output terminals can be converted to corresponding no-load voltage.
- The maximum and minimum values of equivalent no-load voltage are noted for both sides of the transformer, during any of the loading cases (see figure ٤.١).
- The maximum and minimum values of voltage ratio are calculated for each loading case, and the extremes of these ratios are noted. This may not necessarily be the diagonal combinations of overall extreme voltages, because the primary and secondary voltages do not vary in opposite directions to the full extent between light load and heavy load (see figure ٤.٢).
- Using the information on turns ratio, the input current of the primary winding is calculated. The maximum value is noted.

At this stage, the following six necessary parameters have been determined. They frame the electrical design of the transformer:

- maximum primary voltage;
- maximum secondary voltage;
- maximum voltage ratio;
– minimum voltage ratio;
– maximum primary current;
– maximum secondary current.

In order to continue, it is now necessary to know which winding is the tapped winding.

– The rated voltage of the untapped winding will be determined by the maximum voltage on that side of the transformer (see figure 3.c).

– The tapping voltage range on the tapped winding side can then be determined from the correlated voltage and ratio parameters as shown in detail later (see figures 3.c and 3.d). This will often lead to a specification of tappings according to the principle of combined voltage variation (see 3.7 of IEC 67006-1 and further comments below).

– The maximum value of load current for the untapped winding will in principle, together with its rated voltage, define the rated power of the transformer.

Unless otherwise specified, all tappings should be full power tappings (see 3.7 and 3.8 of IEC 67006-1). In order to avoid overdimensioning of the transformer it may, however, be useful to deviate for this main rule. Thus the increase of tapping voltages in the tapped winding may be truncated towards the end of the plus tapping range (combined voltage variation). Likewise the rise of tapping currents towards the minus tapping end is truncated by the application of a maximum current tapping. This means that the tapping power values for these extreme tappings are reduced in comparison with the value in the middle range which is the full rated power of the transformer.

– The principal tapping is selected preferably in the middle of the tapping range. The tapping voltage and current for the tapped winding at this tapping will then be its rated quantities.

– The complete set of parameters is finally collected in a table for the specification.

These different steps will be detailed in line with a practical example in the following subclauses.

\[4] Details of the procedure, step by step

Together with the description of the method in general form, a practical example is demonstrated.

Example:
Transformation from a 115 kV system to a 20 kV system. The primary voltage ranges from 115 kV to the maximum allowable value, \( U_m \), of the system, which is 125 kV. The secondary voltage is required to be 20 kV at no-load, rising to 22 kV no-load voltage to compensate the voltage drop in the transformer at full load current which is specified to be 1000 A. (The actual secondary service voltage is then 19 kV if the voltage drop in the transformer is 1 kV.)

For the sake of simplification, it is assumed from the beginning that the high-voltage winding is the primary winding, which receives active power from a system. This winding is also the tapped winding. The low-voltage winding is untapped. In the following, reference is made to "primary" and "secondary" when this is significant, or to "tapped" and "untapped" when that is significant.

It should therefore be easy to conduct the analysis following the same principle, even if the transformer would be a step-up transformer or if the low-voltage winding is the tapped winding.
Calculation of voltage drop and equivalent no-load voltage

(This is not detailed here, but only referred to for completeness.)

The loading on the secondary side is \((S, \phi)\) at a voltage \(U_r\) on the secondary terminals of the transformer. The corresponding load current is easily calculated. The load also represents an impedance equal to \(Z_L\):

\[
Z_L = \frac{U_r}{S} = \frac{U_r}{S} (\cos \phi + j \sin \phi)
\]

Where \(S\) is in MVA, \(U_r\) is in kV and \(Z_L\) is in ohms. The formula is valid for both single-phase and three-phase loads.

The short-circuit impedance of the transformer may be estimated from similar, existing units. Its value is not very critical for this purpose. Usually, a percentage impedance is estimated. The corresponding value in ohms is obtained by multiplication with the reference impedance of the secondary side of the transformer:

\[
Z_r = \frac{U_r}{S_{ref}} \left( \frac{R_T + j X_T}{U_r + U_r} \right)
\]

The calculation of the voltage drop and corresponding no-load voltage is described in clause 7 of this guide.

Variation range of voltage ratio

Each of the loading cases also contains a statement of the applied primary and secondary side voltages, or range of voltage, at which the load will be drawn. The secondary side voltages are converted to equivalent no-load voltage according to \(8.4.1\). Then the corresponding turns ratio values can be found. The highest and lowest ratios are noted. This is not necessarily \(U_{r, \text{max}} / U_{r, \text{min}}\) and \(U_{r, \text{min}} / U_{r, \text{max}}\), as figure 7.4.b indicates.

\[
n_{\text{max}} = \left( \frac{U_r}{U_r} \right)_{\text{max}}; \quad n_{\text{min}} = \left( \frac{U_r}{U_r} \right)_{\text{min}}.
\]

The extreme ratios indicate the relative width of the required tapping range. If it becomes unusually large, it may be advisable to go back and critically examine the initial assumptions regarding loading cases. At the end of the procedure, the range may be modified to some existing standard notation.

Example (continued)

The extreme combinations of voltages are in this case

\[n_{\text{max}} = 1.47/2.7 = 0.53\text{ and } n_{\text{min}} = 1.07/2.2 = 0.48\]

There are no indications of restrictions for the extreme combinations.
\section*{Rated voltage of the untapped winding}

It is assumed that the high-voltage primary winding will be the tapped winding, and that the low-voltage secondary winding is the untapped winding. Regardless of whether this applies or not, the following procedure, in principle, can now be conducted.

The rated voltage of the untapped winding, $U_{\text{r}}$, will in principle be the highest recognized voltage for any load condition on that winding (applied voltage if it is the primary, equivalent induced voltage if it is the secondary) (see figure 3·c).

IEC 6700·1 states that a transformer shall be capable of continuous service without damage with 5\% over-voltage. This is not meant to be systematically utilized in normal service. It should be reserved for relatively rare cases of service under limited periods of time, for example emergency service or extreme peak loading. If a high-load case used in this analysis is of this character, it may be appropriate to round off the rated voltage figure downwards within this tolerance range.

In North America, the voltage rating is such that

\begin{itemize}
  \item [a)] the transformer is capable of delivering rated output MVA, at a lagging power factor of 8\% or higher with 10\% rated secondary voltage without exceeding the guaranteed temperature rises;
  \item [b)] the transformer is capable of operating at no-load with the primary voltage required for condition a) or 11\% rated voltage, whichever is higher, without exceeding the guaranteed temperature rises.
\end{itemize}

Example (continued)

In our example, the rated secondary voltage (untapped winding) is to be 22 kV.
NOTE – Service voltage ranges on both sides (different scales)

NOTE – Extremes of ratio: Emergency high-load case combines minimum primary voltage with maximum secondary (equivalent no-load) voltage. This is minimum ratio. Combination of $U_{1 \text{ max}}$ with $U_{2 \text{ min}}$ may be unrealistic. Maximum ratio is then instead expressed as $U_{1 \text{ max}}/U_r$, or as $U_r/U_{1 \text{ min}}$. ($U_r$ and $U_1$ have no particular signification).
NOTE – $U_{r\text{, max}}$ will be rated voltage $U_r$, because the secondary winding is untapped. With constant flux voltage variation, the range of tapping voltages on the primary side would be determined by the extreme ratios, multiplied with rated secondary voltage. Rated primary voltage (principal tapping) is selected in the middle.

NOTE – The highest tapping voltage $U_r$, according to figure 3.3c, is unrealistic, maybe even above the highest voltage for equipment, $U_m$, applicable to the primary side system. The range of primary side tapping voltages is therefore truncated at $U_{r\text{, max}}$ from figure 3.3a. The highest plus tappings instead combine with reduced values of $U_r$ ($U_r'$, as indicated in figure 3.3d). This is the combined voltage variation (CbVV) principle (see also figure 3.1).

Figure 3.3c – Determination of tapping range
Range of tapping voltages for the tapped winding

The lowest necessary tapping voltage will be \( U_{1\min} = n_{\min} \times U_{1r} \) (see figure \( \text{\textsection}\cdot\text{c} \)).

The highest tapping voltage may be \( U_{1\max} = n_{\max} \times U_{1r} \).

This is the case if the principle of constant flux voltage variation (CFVV) is followed, but it is not necessarily so. The example in figure \( \text{\textsection}\cdot\text{c} \) shows the case that CFVV would lead to an unrealistically high maximum tapping voltage, maybe even higher than the highest voltage for equipment \( U_m \), applicable for the system. (Technically, there would not be any limitation in terms of core saturation but the overvoltage is not allowed in the system from the point of view of insulation co-ordination.) Maximum ratio cannot then be combined with full rated voltage on the untapped winding. The highest plus tappings can only be used with reduced secondary voltage. The secondary winding tapping voltages are, consequently, noted lower than the rated voltage at the extreme plus tappings, while the primary tapping voltages are kept constant at the maximum applied voltage level (see figure \( \text{\textsection}\cdot\text{d} \)).

This means that the transformer is specified according to the principle of combined voltage variation (CbVV) (see \( \text{\textsection}\cdot2 \) of IEC \( \text{\textsection}\cdot6\text{\textsection}0\text{\textsection}1 \)). The changeover point (see figure \( \text{\textsection}1 \)) is called the "maximum voltage tapping". The tappings beyond that point have reduced tapping power.

The truncated, maximum tapped winding voltage may be expressed with a voltage ratio \( n_u \) in relation to the untapped winding rated voltage:

\[
U_{1\max} = n_u U_{1r} \quad (n_u < n_{\max})
\]

\( n_u \) is \( 321/22 = 321/22 \) = 145.5 kV.

At this stage it is possible to fix the rated voltage of the primary winding in the middle of the range of ratio from \( 482.4 \) to \( 715.0 \), say \( 650.7 \). Then \( 650.7 \times 22 = 1431.5 \) kV.
Before proceeding, it should be noted that rated current and rated power are defined as continuous duty. The continuous duty rated power is used as the general reference for impedance and losses (see §1.1 and §1.3 of IEC 67006-1). It is also related to the limitations of temperature rise in steady state. The actual loading, on the other hand, is usually variable over the day and over the year. Temporary loading beyond rated power is possible and may affect the required voltage regulation range of the transformer.

The problem of how to convert a time-variable load to an equivalent continuous load is treated in IEC 60306 for oil-immersed transformers and in IEC 60906 for dry-type transformers.

The required continuous load power represents a specific value of load current. This value is higher when the service voltage is low and the transformer should be dimensioned accordingly. The highest value occurring in the untapped winding for any loading case is the rated current of this winding. (In the example, it is the secondary low-voltage winding.)

This rated current, together with the rated voltage, determines the value of rated power, \( S_r \). The calculated value will be suitably rounded off at this stage.

**Example (continued)**

The secondary load current was from the beginning specified as \( 1000 \) A, regardless of the output service voltage of the transformer. This immediately becomes the rated current of the (untapped) secondary winding.

The rated current of the primary tapped winding is calculated by the ratio at the principal tapping:

\[
\frac{1000}{705.5} = 1.418 \text{ A.}
\]

But this is not the dimensioning current for the primary winding. The highest tapping current occurs at minimum tapping voltage, in the combination of \( 10.5/22 \) kV: ratio \( 4.68 \).

Therefore \( I_{\text{max}} = \frac{1000}{4.68} = 214.3 \text{ A.} \)

### Optional maximum current tapping, reduced power tappings

In principle, the rated power will apply over at least a considerable part of the tapping range. In the tapped winding, this means that tapping currents vary inversely with the tapping voltages. Towards the low-voltage end of the tapping range, this would sometimes result in rather high tapping current values, which represent unwanted overdimensioning of the whole winding. It may then be decided to truncate the variation at a certain point, called the maximum current tapping. From there on, the tapping current values are kept constant in the tapped winding and the corresponding values for the untapped winding will taper off accordingly. These tappings then have reduced tapping power (see figure 23).

The tapping voltage ratio at this point is \( n_i \):

\[
U_{\text{r},i} = n_i \times U_{\text{r}} \quad (n_{\text{min}} < n_i < n_{\text{u}} < n_{\text{max}})
\]

The tappings above the maximum voltage tapping where the flux density will be reduced in service are also reduced power tappings.
Example (continued)

This transformer is specified according to the principle of combined voltage variation. The highest plus tappings have truncated primary tappings voltages at \(321\) kV, and the secondary tapping voltages will therefore decrease for tappings above the changeover point. These tappings will be reduced power tappings, because the tapping current of the winding stays constant at \(1\) kA.

The option of using a maximum current tapping in the minus tapping range (limiting the tapping currents of the primary winding) is not utilized in this example.

\[
\begin{align*}
U_A & \quad \text{plus tapping range} \\
U_B & \quad \text{minus tapping range} \\
I_A & \quad \text{constant, not rising, for tappings below the maximum current tapping} \\
I_B & \quad \text{constant, not rising, for tappings above the changeover point} \\
S_{AB} & \quad \text{tapping factor}
\end{align*}
\]

NOTES

1. Subscript \(A\) refers to the tapped winding. Subscript \(B\) refers to the untapped winding.

2. The changeover point from CFVV (constant flux) to VFVV (variable flux) is shown in the plus tapping range. It constitutes a maximum voltage tapping \(U_A\) constant, not rising, for tappings above the changeover point.

3. An additional, optional breaking point, at maximum current tapping, is indicated in the minus tapping range \(I_A\) constant, not rising, for tappings below the maximum current tapping.

Figure 24 – Illustration of combined voltage variation (CbVV) with a maximum current tapping (in the minus range)
Completing the table of tapping quantities, selection of the principal tapping

At this stage, the maximum and minimum voltage ratio has been found. Between these are two ratios, at which the changeover points of tapping voltage and tapping current in the tapped winding are placed. Throughout the range between the changeover points, the rated power applies. Outside this range, the tapping power tapers off towards both ends.

The principal tapping is placed within the central full-power range of tappings, and preferably at the mid-point of the complete tapping range.

The tapping voltage and tapping current of the tapped winding at the principal tapping are thereby elevated to be rated quantities for the tapped winding.

Example (continued)
The specification of the principal tapping of the primary winding in the middle of the complete tapping range was already made at ratio $\frac{705.5}{51.121}$, $\pm 1.5\%$ 22 kV.

The skeleton of the tapping table is now completed. It remains to adjust and complete it to fit in with the standard number of steps available on the tapchanger for some standard set of tapping percentages.

Summarizing, the voltages and currents in a CbVV regulated transformer are as set out in table 4.

<table>
<thead>
<tr>
<th>Tapping</th>
<th>Ratio</th>
<th>Tapping voltage</th>
<th>Tapping currents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plus</td>
<td>$n_{\text{max}}$</td>
<td>$U_{1\text{max}}$</td>
<td>$U_{2r}$</td>
</tr>
<tr>
<td></td>
<td>$n_u$</td>
<td>$U_{1\text{max}}$</td>
<td>$U_{2r}$</td>
</tr>
<tr>
<td>Principal</td>
<td>$n_f$</td>
<td>$U_{1r}$</td>
<td>$U_{2r}$</td>
</tr>
<tr>
<td>Minus</td>
<td>$n_{\text{min}}$</td>
<td>$U_{1\text{min}}$</td>
<td>$U_{2r}$</td>
</tr>
</tbody>
</table>

Example (continued)
For the transformer in the example, the resulting rounded values suitable for an order enquiry are:
- power: 38 MVA
- voltage ratio: $111 \pm 1.5\% \times 22$ kV
- regulation: CbVV; max. voltage 143 kV

The parameters of the actual transformer which has been considered in this subclause can likewise be tabulated as shown in table 5. This table also indicates a possible end result by which the transformer can be specified, i.e. with a negligible change to the principal tapping voltage and the use of a 16 step, 17 position tap-changer.
### Table 9 – Transformer parameters

<table>
<thead>
<tr>
<th>Ratio/tapping factor %</th>
<th>Voltages kV</th>
<th>Currents A</th>
<th>Power (MVA)</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HV</td>
<td>LV</td>
<td>HV</td>
<td>LV</td>
</tr>
<tr>
<td>2,100/1,110</td>
<td>123</td>
<td>122,1</td>
<td>100,1</td>
<td>94,1</td>
</tr>
<tr>
<td>3,091/1,110</td>
<td>123</td>
<td>178,9</td>
<td>100,1</td>
<td>38,1</td>
</tr>
<tr>
<td>3,090/1,110</td>
<td>141,4</td>
<td>181,4</td>
<td>100,1</td>
<td>38,1</td>
</tr>
<tr>
<td>2,869/881</td>
<td>10,7</td>
<td>20,2,6</td>
<td>100,1</td>
<td>38,1</td>
</tr>
</tbody>
</table>

\[ \text{Maximum voltage tapping} \]

\[ \text{Principal tapping} \]

### Convertor applications with standard transformers

Clause 1 of IEC 61730-1 states that transformers for static converters are special and are exempt from the general category of power transformers for which IEC 61730-1 is applicable in full. See IEC 8731-1.

This clause indicates the precautions which should be taken when moderate size, general purpose catalogue standard power transformer are to be used for convertor applications. There are two considerations:

- influence of distorted voltage;
- influence of distorted current.

### Influence of distorted voltage

A transformer feeding a convertor circuit from a public power system has an applied voltage with negligible distortion (with regard to influence on core loss and core heating).

A transformer with its primary energized from an inverter circuit may have a quite complex voltage waveshape, varying from time to time. As a general statement, the waveshape of the flux in the core, being the integral function of the applied voltage, is less distorted than the voltage waveshape. The essential voltage parameter which determines the amplitude value of flux density is the average value, not the r.m.s. value, of voltage. The actual waveshape of voltage should be submitted with an enquiry for this kind of application. De-rating of the transformer with respect to voltage is, however, usually not necessary.

Many invertor circuits work with a frequency different from that of the supply system, sometimes even with variable frequency. This should of course be mentioned in the enquiry.

A special problem may be asymmetry of the applied waveshape from the inverter due to imperfect firing control. This may lead to saturation because of a d.c. component.
1.7 Influence of distorted current, general

A distorted current waveshape will give rise to increased additional loss in the transformer due to eddy loss in the windings and stray losses in metallic mechanical parts. Both kinds of loss are related to the square of the time derivative of the leakage flux, and the leakage flux, in turn, is proportional to winding current. Another effect of the distorted current is a higher audible noise level from the transformer containing more sound of higher frequencies.

The increase of additional loss causes higher average temperature rise. This may be objectionable, *per se*, but there is also a risk of damage due to a more pronounced rise of local hotspot temperatures. This is dealt with in the following subclauses.

1.8 Overall eddy losses in windings

Calculation of eddy losses in the winding are usually made with the aid of the harmonic spectrum which should preferably be submitted in the enquiry.

For windings made with round or rectangular conductors, the specific eddy losses from harmonics increase with the square of the frequency. The contribution from the \( j \)th harmonic, having an r.m.s. value of \( I_j \) amperes, may thus be written:

\[
P_{ej} = \text{const} \times (j \times I_j)^2
\]

The total eddy loss \( P_e \) in the winding from the whole current spectrum may then be expressed as a multiple of the eddy loss for the fundamental power frequency, \( P_{e1} \), which is obtainable by conventional calculation. Typical values of \( P_e/P_{e1} \) are in the range of 2 to 3, when the load is a convertor with relatively well smoothed d.c. current. Three-phase convertor circuits in principle only contain certain harmonics. Higher harmonics are reduced because of the rounding off of the ideal waveshape through the commutation reactance. For further information, see literature on convertor circuits.

The theory and estimate given above relate to windings of round or rectangular conductors only. Transformer windings of full-width foil present a current concentration towards the edges of the foil windings and therefore the theory is more complicated. If such a transformer should be applied for convertor loading, the manufacturer should be requested to confirm that the design is suitable.

1.9 Stray losses in mechanical parts

These losses are more difficult to predict than winding eddy losses. They occur in ferromagnetic material (core-steel or structural steel), and the penetration is a non-linear phenomenon which does not lend itself properly to superposition analysis. Stray losses in mechanical parts are of importance in large and special convertor transformers, but do not usually play any great part when a small conventional transformer is used for convertor load.
9.0 Combined additional loss, possible de-rating

The combined additional loss (eddy loss in windings plus stray loss in mechanical parts) at sinusoidal rated current may be determined after the routine short-circuit test by measuring the loss at several frequencies. This procedure will identify that portion of the loss which is dependent on the square of the frequency and that portion of the loss which is dependent on frequency to a power above unity. The proportion between the two depends on the design.

The combined additional loss to be expected for a converter waveshape may typically be in the range of \( \frac{1}{10} \) to \( \frac{1}{7} \) times the corresponding loss determined with sinusoidal current during the routine test of the transformer.

From the point of view of average temperature rise, it may be advisable to restrict the allowable continuous current in service slightly below rated current, so that the total loss, \( I^2R \) plus additional loss, remains no higher under converter load than with rated sinusoidal current.

9.7 Local hotspots

The eddy loss intensity varies from point to point in the winding, depending on local intensity and direction of the magnetic stray field. The intensity in parts close to the main duct between the primary and secondary winding, is typically three times as high as the average value. Still higher intensities may prevail locally.

The average temperature rise of a transformer winding above the average of the surrounding cooling medium (oil or air) has a direct influence on the life of its insulation system. This temperature differential (winding gradient) is proportional to winding (losses) in a typical self cooled transformer. It is recognized that the local hot-spot temperatures may be as much as 10 °C and 50 °C hotter than the average winding temperatures in oil filled and dry type transformers respectively under rated sinusoidal loading conditions.

The average winding gradients are normally measured under sinusoidal loading conditions.

The eddy and stray losses in a transformer can significantly increase while supplying harmonic loads in converter applications. This can result in increased local hotspot temperatures and therefore excessive loss of life on a transformer.

It is important that the expected harmonic content of the converter load be provided while specifying a transformer for this application.

10. Guide to the measurement of losses in power transformers

10.1 Test results, guarantees, tolerances, uncertainty limits

The test clauses in the various parts of IEC 6700 contain rules as to how the original measurements shall be evaluated and corrected when the test has been made under conditions differing from reference conditions or with test quantities differing from specified target values.

Clause 9 of IEC 6700 deals with tolerances, acceptable deviations of certain guaranteed parameters of the transformer which are to be verified in testing.
When the test result is expressed as a numerical quantity (and not only as a verdict of withstanding a test procedure or not), it is not an exact number but suffers from uncertainty. How wide this margin of uncertainty is depends on the quality of the test installation, particularly its measuring system, on the skill of the staff and on measurement difficulties presented by the test object.

The submitted test result shall contain the most correct estimate that is possible, based on the measurements that have been carried out. This value shall be accepted as it stands. The uncertainty margin shall not be involved in the judgement of compliance for guarantees with no positive tolerance or tolerance ranges for performance data of the test object.

However, a condition for acceptance of the whole test is that the measurements themselves have to fulfil certain requirements of quality. Statements of limits or uncertainty shall be available and these statements shall be supported by a documented traceability (see ISO 17025).

Traceability, quality aspects on measuring technique

Traceability of measurements means that a chain of calibrations and comparisons have been carried out, through which the validity of the individual measurement can be traced back to national and international standards of units preserved in recognized institutions of metrology. Evidence of such traceability should contain the following items.

a) Certified information about errors (amplitude errors and phase angle errors) of the components of the measuring system (transducers for voltage, current and power, voltage dividers and shunts, indicating or recording instruments, etc.)

This may comprise:
- certificates from the manufacturers of individual components;
- certification from calibrations carried out at independent precision laboratories;
- certificates of calibrations made in the plant by means of precision instrumentation and specialist staff brought there for that purpose;
- direct comparisons of the test room installation with a complete precision measuring system (overall system calibration).

Subclauses 3.01 and 4.01 below indicate the particular importance of phase angle errors for load loss measurement due to the very low power factor of the short-circuited transformer. The phase angle errors of conventional, reconnectible voltage and current transformers depend on the combined instrument impedance burden and vary with the actual voltage or current value across each measuring range. This makes it difficult to sort out unknown systematic errors (which may be corrected for) from unknown systematic and random errors which cannot be eliminated in the particular case.

b) Information on the quality of the test power source: voltage harmonic content, stability of voltage and frequency

The test clauses in IEC 6700-1 contain certain limitations on voltage waveshape in general and particularly for the testing of transformers at no load. These requirements lead to consequential requirements on the test supply, its internal impedance and connection. This has to be known and accounted for. If direct supervision of the waveshape during individual tests is not carried out, systematic information from the special studies should be available.

c) Information about the test environment in terms of electrical disturbance (electromagnetic fields, earthing, shielding)
This would be based on investigations looking for residual random noise and erroneous signals entering into the measuring system by stray capacitances or by magnetic induction or voltage drop in leads or cable sheaths possibly carrying circulating earth current. The investigation would typically involve running a dummy test with actual test power but opening or short-circuiting signal entrances to the measuring system, moving or turning around components, and applying additional shielding or earthing in an intelligent and systematic manner.

d) Analysis of systematic errors and measuring uncertainty for the particular type of test under consideration, based on items a) to c) above. This analysis should contain a rational analytical procedure, taking into account the interaction and combination of different sources of systematic error and random uncertainty.

The analysis should distinguish between random uncertainty and systematic error, and make a statement as to how different individual errors are combined, quadrature combination of different effects (root of sum of squares – RSS), or linear addition (maximum possible error).

The analysis should give the following details:
- the range of the measuring capabilities of the test system;
- the instrument settings used for the measurement;
- an example of test data evaluation with the correction procedure used corresponding to the instrument settings.

e) The test department shall possess routines for continuously maintaining the quality of measurements. This should be by regular checking and calibration routines for components and for the complete system. It may comprise both in-house functional comparisons between the alternative systems, checking the stability and periodical recalibration of components as indicated above under item a).

10.3 Fundamental sources of error in power transformer load loss measurement

The method for load loss determination is laid down in \(10.4\) of IEC 67006-1 as follows.

"The short-circuit impedance and load loss for a pair of windings shall be measured at rated frequency with approximately sinusoidal voltage applied to the terminals of one winding, with the terminals of the other winding short-circuited, and with possible other windings open-circuited."

The load loss is represented as the resistive part of the series impedance of the equivalent circuit of the transformer. The series impedance is regarded as linear. This means that the load loss is supposed to vary with the square of the current. The measurement of test current is a contributing source of error for the determination of the loss.

The variation of load loss with temperature is also considerable. Annex E of IEC 67006-1 prescribes how \(I^2R\) losses and additional losses shall be treated when load losses are to be recalculated for the reference temperature of the winding. Errors in the previous resistance measurement and in the assessment of winding temperature during the loss measurement therefore contribute to the error of the evaluated loss value, valid at the reference temperature. Warnings are given in \(10.4\) of IEC 67006-1.

The series impedance of the transformer is mainly inductive. The power factor of the impedance tends to fall with rising values of rated power.
A typical example:

a) 1 000 kVA transformer: load loss 1% of rated power, short-circuit impedance 6% of reference impedance – power factor of the series impedance, consequently, 761.0;

b) 0.1 MVA transformer: load loss 4%, short-circuit impedance 51 – power factor 720.0.

During the test, voltage, current, and active power are measured using measuring systems which contain transducers for matching the high values of voltage and current to the measuring instruments. Conventional systems include inductive-type measuring transformers for voltage and current to feed indicating instruments of electromechanical type. Measuring systems of more recent design employ special two-stage or zero-flux current transformers, capacitor voltage dividers, blocking amplifiers, electronic digital multiplier wattmeters, etc. Different types of measuring systems present different characteristics with regard to measuring errors, but some observations of principle are generally valid. The loss to be measured is, by definition, \( P = U \times I \times \cos \phi \).

The composite relative error is obtained by taking the natural logarithm of both members and then differentiating:

\[
\frac{\partial P}{P} = \frac{\partial U}{U} + \frac{\partial I}{I} - \frac{\sin \phi}{\cos \phi} \partial \phi
\]

The phase angle \( \phi \) between the phasors of voltage \( U \) and current \( I \) is close to \( \pi/2 \) (90° inductive). The power factor \( \cos \phi \) is a small number. We rewrite the coefficient in front of \( \partial \phi \):

\[
- \frac{\sin \phi}{\cos \phi} = - \left( \frac{1 - \cos \phi}{\cos \phi} \right) \approx - \frac{1}{\cos \phi}
\]

![Figure 33a – Phasor diagram indicating measuring error](image).

![Figure 33b – Relative error as a function of power factor](image).

![Figure 38 – Relative error of measurements](image).
This is a number much larger than unity, which indicates that a certain relative error in the estimation of the phase angle (in radians) results in a much larger relative error in the estimation of the loss, while relative errors in voltage and current amplitudes contribute without magnification. Figure ττa attempts to illustrate this. The envelope of uncertainty of loss estimates as a function of the power factor of the transformer impedance will therefore have the general shape indicated in figure ττb.

The central problem for the measurement of load losses in power transformers is therefore how to reduce, or correct for, phase shift in the complete measuring system or in its individual components.

Phase angle error of a conventional loss measuring system, possibility of correction

The conventional measuring system consists of magnetic type voltage and current transformers and an electrodynamic wattmeter.

The measuring transformers have phase angle errors $\delta_u$ and $\delta_i$ respectively, expressed in radians. The inductance of the wattmeter coil introduces a phase lag between the applied voltage from the voltage transformer and the current in the voltage coil of the instrument. The numerical value of this small phase shift (again in radians) is $\omega L/R$, where $L$ is the inductance of the voltage coil and $R$ is the total series resistance in the instrument and possible external resistor boxes. This wattmeter phase shift is denoted $\delta_{\omega}$.

If the original phase angle between the voltage and current in the test object is $\phi$, the actual phase angle in the wattmeter will be

$$\phi' = \phi + \delta_u - \delta_i - \delta_{\omega} = \phi + \delta\phi$$

If the total phase angle error $\delta\phi$ is positive, then the estimated power factor, $\cos \phi'$, is lower than the correct value, $\cos \phi$. A correction of measured loss will have a positive sign:

$$P = P' \left( \frac{1}{1 - \delta\phi / \cos \phi} \right) \approx P' \left( 1 + \frac{\delta\phi}{\cos \phi} \right)$$

Conventionally, phase angle errors are expressed in submultiples of electrical degrees:

$$1 \text{ min} = \frac{\pi}{180 \times 1.6} = \frac{\pi}{1.981} \text{ rad}$$

Example:

A transformer has a $\cos \phi = \cdots \tau$. A total phase angle error of $\tau \text{ min}$ then results in a relative error of nearly $\tau \%$ in measured loss:

$$\frac{\tau \times \frac{\pi}{1.981}}{1 \times \cdots \tau} = \tau \times \frac{\pi}{1.981} \approx \tau \%$$

The phase angle errors caused by the voltage and current transformers are, in practice, difficult to assess correctly. A calibration certificate is normally supplied with a good laboratory type measuring transformer but it cannot satisfactorily cover the variable parameters of the circuit (measuring range, value within the range, instrument burden). The conventional degree of precision submitted in the calibration certificate is also usually relatively limited. The available corrections for the known systematic errors are to be applied. But there still remains an unknown systematic error which cannot be corrected.
Another difficulty arises from the fact that when analogue type instruments are used, the wattmeter deflection will often be only a small fraction of the scale, even though a special low power factor instrument is employed. The relative random uncertainty of the reading is therefore so large that it may overshadow small, known systematic errors. The validity of a corrected value (with known systematic errors removed) will then be no better than that of the uncorrected reading.

All this taken together means that when a conventional measuring system of the type described has been used for a test, it is very difficult to state a reliable correction which would bring the result up to a higher degree of accuracy. The measurement cannot readily be traced back to standards unless a direct calibration would be performed with the specific setting of the complete measuring system that has been employed for a particular test. Otherwise, the estimate of possible uncertainty should be made quite conservatively, based on overall limits for the individual components of the system and the uncertainty of the observation of the instrument readings.

Advanced measuring systems

An advanced measuring system is understood to be a system containing two-stage or zero-flux current transducers, usually capacitor-type voltage divider circuits, electronic blocking amplifiers and adjustable error compensation circuits, digital electronic power transducers.

It is a characteristic of such a system that individual components should be adjusted and calibrated against standards of high precision so that their systematic errors will become negligible in comparison with the remaining random uncertainty range. Instrument loading is eliminated as a source of error because of the output amplifiers.

The resulting phase angle error for the complete system may be of the order of \( \frac{0.1}{1000} \mu \text{rad} \) to \( \frac{0.2}{1000} \mu \text{rad} \) (0.01 min to 0.1 min). With such systems, an overall maximum error of \( \pm 3 \% \) may be achieved for the loss determination down to a power factor of 20 or even lower.

An overall calculation of uncertainty is made which is valid either for the whole range of test object data or for the individual range settings of the components. The calibration is to be maintained by regular checks e.g. against another portable system used for this purpose only.

The low signal power levels appearing inside advanced measuring systems makes it particularly important that the measuring system is meticulously checked against disturbance from electromagnetic fields etc. at the time of installation (see 2.0.2 c).

Measurement of no-load loss

The measurement of no-load loss differs from measurement of load loss for the same transformer in that the power factor is considerably higher, and that the test current is heavily distorted.

The no-load loss is, in principle, referred to undisturbed sinusoidal voltage on the transformer terminals. Subclause 5.1 of IEC 67006-1 gives a criterion for a satisfactory waveshape based on a comparison between the readings of two voltmeters, sensing average value and r.m.s. value, respectively. This implies requirements not only on the no-load waveshape of the test supply but also on its internal impedance because of the difficult current waveshape and on its connection.
The no-load current and power rise rapidly with the applied voltage. The measurement and adjustment of voltage is therefore critical and contributes implicitly to the uncertainty of the loss determination.

It is further specified in IEC 67006-1 that the connection of a three-phase transformer shall be made in such a way that the voltages applied across the different phases of the winding be as nearly sinusoidal as possible.

The most difficult cases, both with regard to test power requirements (voltage waveshape distortion) and power measurement, usually arise when testing large single-phase transformers.

Finally, the measured no-load loss is sensitive to the prehistory of magnetization. Remanence in the core after saturation during winding resistance measurement with d.c., or by unidirectional long-duration impulses, may leave a trace in the results. A systematic demagnetization of the core before the no-load measurements is sometimes practised to establish representative results.
Annex A
(informative)

Basic relations for single-phase and two-phase earth faults

Before the fault, there is a symmetrical system service voltage

\[ U = U^+; \quad U^- = U^- = 0 \]

When the fault is established, the component voltages become \( V^+, V^-, V' \).

The short-circuit impedances of the whole system, as viewed from the fault, are \( Z^+, Z^-, Z' \).

The superposed fault current components are \( I^+, I^-, I' \).

The following relations apply:

\[
\begin{align*}
U - V^+ &= I^+ Z^+ \\
-V^- &= I^- Z^- \\
-V' &= I' Z'
\end{align*}
\]

(A.1)

By definition, phase voltages and currents are

\[
\begin{align*}
V_A &= V^+ + V^- + V' \\
V_B &= \alpha^* V^+ + \alpha V^- + V' \\
V_C &= \alpha V^+ + \alpha^* V^- + V'
\end{align*}
\]

(A.2)

\[
\begin{align*}
I_A &= I^+ + I^- + I' \\
B &= \alpha^* I^+ + \alpha I^- + I' \\
I_C &= \alpha I^+ + \alpha^* I^- + I'
\end{align*}
\]

(A.3)

Case 1 – Single-phase earth fault on phase A

\[
I_B = I_C = 0 \quad \text{(A.4)}
\]

\[-I' = \alpha^* I^+ + \alpha I^- = \alpha I^- \]

\[= I^+ = I^- = I' = l \quad \text{(A.5)}
\]

\[V_A = 0 \quad \text{(A.6)}
\]

\[= V^+ + V^- + V' = 0 \quad \text{(A.7)}
\]
Combine equations (A.6) and (A.7) with the sum of the equations (A.1):

\[ i^+ = i^- = i^' = \frac{U}{Z^+ + Z^- + Z^*} \]  \hspace{1cm} (A.8)

**Case Y – Earth fault on phases B and C**

\[ V_B = V_C = \cdot \]
\[ -V^* = a^* V^+ + a V^- = a V^+ + a^* V^- \]
\[ \therefore V^+ = V^- = V^' = V \]  \hspace{1cm} (A.8)

According to equation (A.1)

\[ i^+ = \frac{U}{Z^+} - \frac{V}{Z^+} \]
\[ i^- = -\frac{V}{Z^-} \]
\[ i^' = -\frac{V}{Z^*} \]

\[ i_A = i^+ + i^- + i^' = \frac{U}{Z^+} - V \left( \frac{1}{Z^+} - \frac{1}{Z^-} + \frac{1}{Z^*} \right) = \cdot \]

\[ \therefore V = \frac{U}{Z^+} \times Z \]

and

\[ \frac{1}{Z} = \left( \frac{1}{Z^+} + \frac{1}{Z^-} + \frac{1}{Z^*} \right) \]  \hspace{1cm} (A.9)

\[ i^+ = \frac{U}{Z^+} - \frac{V}{Z^+} = \frac{U}{Z^+} \left( -\frac{Z}{Z^+} + \frac{1}{Z^+} \right) = \frac{U}{Z^+} \left( \frac{U \times Z}{Z^+} \right) \]
\[ i^- = \frac{U \times Z}{Z^+ \times Z^-} \]
\[ i^' = \frac{U \times Z}{Z^+ \times Z^*} \]  \hspace{1cm} (A.10)